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UNIVERSITAT JAUME I

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Economics Department

Dimitrios Xefteris
Iván Barreda-Tarrazona
Aurora García-Gallego
Nikolaos Georgantzís

2018 / 08

Castellón (Spain)

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Dimitrios Xefteris
University of Cyprus
Economics Department
xefteris.dimitrios@ucy.ac.cy

Iván Barreda-Tarrazona
Universitat Jaume I
LEE & Economics Department
ivan.barreda@uji.es

Aurora García-Gallego
Universitat Jaume I
LEE & Economics Department
mgarcia@uji.es

Nikolaos Georgantzís
Burgundy School of Wines and Spirits Business & UJI
LEE & Economics Department
nikolaos.georgantzis@bsb-education.com

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Abstract

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Keywords: Catalog competition; Hotelling; mixed equilibrium; experiment

JEL classification: C72, C92, D43, D90

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Equilibrium characterization and experimental evidence*

Dimitrios Xefteris† — University of Cyprus

Iván Barreda-Tarrazona‡ — Universitat Jaume I

Aurora García-Gallego § — Universitat Jaume I

Nikolaos Georgantzís¶ — Burgundy School of Wine and Spirits Business

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*This work was supported by the Spanish *Ministerio de Economía y Competitividad* (grant ECO2015-68469-R AEI/FEDER) and *Universitat Jaume I* (grant P1-1B2015-48). We would also like to thank participants at the Jornadas de Economía Industrial 2015 in Alicante (Spain), the ASSET 2016 in Thessaloniki (Greece) and the EARIE 2018 in Athens (Greece) for their helpful comments and suggestions. The usual disclaimer applies.

†Corresponding author: Economics Dept., University of Cyprus; P.O. Box 20537, CY-1678 Nicosia, Cyprus; e-mail: xefteris.dimitrios@ucy.ac.cy.

‡LEE & Economics Dept., University Jaume I, 12071 Castellón, Spain; & CERME, Management and Economics Depts., University Ca'Foscari, Venice, Italy; e-mail: ivan.barreda@eco.uji.es.

§LEE & Economics Dept., Universitat Jaume I; Avda. Sos Baynat s/n, 12071-Castellón, Spain; e-mail: mgarcia@uji.es.

¶Burgundy School of Wine and Spirits Business, France & LEE, Universitat Jaume I, Spain; 29 Rue Sambin-BP50608, 21006-Dijon CEDEX, France; e-mail: nick.georgantzis@gmail.com.

1 Introduction

Hotelling's (1929) first-location-then-price game is the cornerstone of the literature of product differentiation. In this celebrated model the product characteristics' space is modeled by means of a linear city where two competitors set-up their shops and where consumers' residences are assumed to be uniformly distributed. Considering: a) that each consumer buys one unit of good; and b) that the choice from which shop to buy this unit of good is made on the basis of proximity to the consumer's residence (preferences for product characteristic) and the price that the shop charges, this model truly captures the interaction between price and product characteristics' choices and, thereafter, the effect of this interaction on the determination of equilibrium outcomes (prices, degree of product differentiation, firms profits and social welfare). Hotelling's (1929) idea to represent the product characteristics' space by a linear segment was indeed ingenious since it captures the basic trade-offs that consumers with different tastes face when they need to choose between differentiated products.

As far as the timing of these choices is concerned, the Hotelling model –and most of its variants– assumes that competitors *first* decide where to set up their shops and *then* –after their location choices become common knowledge– what price to charge for the product that they sell. This timing of choices fits certain cases of firm competition but it is certainly not relevant for many others. Consider, for example, the common case of two firms which have to reveal their new products in a certain exhibition: firms simultaneously announce both product characteristics and prices. In fact, competition in oligopolistic industries such as cars or smart-phones takes place clearly in this simultaneous form. Hence, Hotelling's (1929) timing assumptions are not necessarily a good fit to some cases of real world oligopolistic competition.

Indeed, the idea that simultaneous choice of product characteristics and price might better describe certain frameworks of real world oligopolistic competition has been present in the literature for a long time; and it is by no means ours. In fact, it is first encountered in Lerner and Singer (1937) critique of Hotelling's (1929) work. They specifically argue that, when the competition is between player A and player B, player B should *take both A's location and his price as fixed in choosing his own location and price.* Dasgupta and Maskin (1986) and Economides (1984, 1987), many years later, discussed some properties of this more intuitive simultaneous model of product differentiation. Following Monteiro and Page (2008) and Fleckinger and Lafay (2010) we use the term *catalog* competition to distinguish this game from the two-stage Hotelling model.

A catalog consists of a product characteristic and of a price and, hence, in the framework of the linear city model it is just a pair of a shop location and the price that it charges. It is easy to show that such a catalog game admits no equilibria in pure strategies when each consumer is assumed to always buy exactly one unit of good. Dasgupta and Maskin (1986), however, were the first to provide formal conditions which ensure that this catalog game has an equilibrium in mixed strategies. Monteiro and Page (2008), moreover, proved that a large family of catalog games admits equilibria in mixed strategies and characterized a family of such catalog games which includes the one-stage variation of the Hotelling game that this paper studies.

Despite the fact that we already have these existence proofs, we know nothing regarding the nature of equilibria of catalog competition games. This is because characterizing a mixed strategy equilibrium in a catalog game is not a straightforward task: a mixed strategy in this framework involves a probability distribution with a *multidimensional* support. That is, unlike the price subgames of the two-stage Hotelling game (Osborne and Pitchik 1987), the all-pay auctions (Baye et al. 1996), Downsian competition with a favored candidate (Aragonès and Palfrey 2002) or other games which admit equilibria only in mixed strategies whose underlying probability distributions have unidimensional support (the support of these mixed strategies is a subset of $\mathbb{R}$), the equilibrium of a catalog game involves mixed strategies with two-dimensional support (their support is a subset of $\mathbb{R}^2$). This additional dimension, obviously, complicates matters in several degrees of magnitude and makes any characterization attempt intractable when considering the model in its general form.

This paper attempts to shed light on the nature of equilibria of catalog games by considering a variation of the model with a discrete set of locations (available product characteristics) and continuous pricing. Discretization of elements of continuous games in order for a mixed equilibrium to be identified is not uncommon in spatial competition literature.[1] That is, we consider that two firms compete in

[1] See, for example, Aragonès and Palfrey (2002) and Hummel (2010).

catalogs (locations and prices at the same time) but, compared to the standard setup in which firms locate at a point in the unit interval we focus on a case in which firms can locate to the western, to the central or to the eastern district of the linear city. We have to stress here, though, that despite the fact that the set of locations in our setup is finite, the set of available prices is not. Therefore, since the strategy space of each firm is the Cartesian product of these two sets, we have that the strategy space of each firm is infinite. This, along with the fact that the game is not a constant-sum game and that firms' payoff functions exhibit discontinuities, implies that existence of a unique symmetric equilibrium and, thereafter, possibility of full characterization are not guaranteed by any known theorem.

We are able to fully characterize an equilibrium and to, moreover, prove that it is the unique symmetric equilibrium of the game. Given that this equilibrium is in mixed strategies it is necessary that we have in mind that its nature depends on the exact level of competitors' risk-aversion. We are able to characterize this mixed equilibrium considering that a firm's payoff function is any increasing function of her profits. That is, we can study how equilibrium strategies change when the risk-aversion level of the competitors varies. We find that some of its qualitative characteristics are robust to variations in the risk attitudes of the two competitors. The location which is more probable for a firm to locate at is the central one, and the prices that firms charge are never close to zero. Moreover, the price that a firm which locates at the periphery of the city (that is, at the western or at the eastern district) charges is larger, in expected terms, than the price of a centrally located firm. In other words, firms more often produce mainstream varieties and the prices of those mainstream varieties are found be lower than the prices of specialized ones.

The existence and full characterization of a unique equilibrium in mixed strategies for any possible degree of risk aversion of the two competitors allows us conduct an experiment which can credibly test the hypothesis that actual agents behave according to a mixed equilibrium theoretical prediction.[2] In

[2]Models of spatial differentiation have been tested by several experimental studies. For example, Brown-Kruse and Schenk (2000), Collins and Sherstyuk (2000), and Huck et al. (2002), study experimental spatial markets with two, three and four firms, respectively. In all these papers, subjects' only decision variable is location. Therefore, like in Brown-Kruse et al. (1993), price is an exogenously given parameter. On the contrary, Camacho-Cuena et al. (2005) extend the basic framework considering also endogenous consumer locations. Finally, Orzen and Sefton (2008) study price dispersion as predicted by mixed strategy equilibria, considering exogenous firm locations while Barreda et al. (2011) experimentally study the location-then-pricing framework.

most non-trivial symmetric games with only mixed equilibria (or with only mixed symmetric equilibria) the underlying probability distributions of the mixed strategies are almost always computed under the assumption that agents are risk-neutral. Examples of such experimental studies are Arad and Rubinstein (2012), Collins and Sherstyuk (2000) and Aragonès and Palfrey (2004).[3] The theoretical predictions of the model provide a unique opportunity to test whether agents use Nash equilibrium strategies in a game with only mixed equilibria. Furthermore, we are particularly interested in studying the effects of subjects' risk aversion, as elicited through a separate task, on price-setting behavior, thus testing the predictive power of a central element of the theoretical framework.

Based on a within-subject design with two independent tasks, our experiment is designed to test the predictions of the catalog competition model, especially regarding the role of subjects' risk aversion on pricing behavior. In contrast, the impact of risk attitudes on location decisions is more difficult to assess, given that equilibrium and out-of-equilibrium behavior is predicted to yield opposite effects. Finally, using a between-subject, two-treatment design, we are the first to experimentally compare simultaneous location and pricing choices to those obtained under the standard first-location-then-pricing game. Overall, our results confirm the theoretical predictions. In particular, central locations are more often observed than extreme ones and prices are higher in the extremes than in the centre. Also, risk aversion is found to have the predicted negative effect on prices given a location. Finally, although location profiles are similar across treatments in terms of the competitive pressure they imply on prices, the latter are less extreme in catalog than in sequential competition.

Apart from a confirmation of the theoretical predictions, our experimental results are interesting for a number of reasons. First, the successful use of subjects' risk aversion elicited through an independent task as an explanatory variable for pricing behavior confirms the relevance of risk attitudes on price competition. This is by no means a trivial finding, given the ongoing debate on the effects of risk attitudes on competition or cooperation. In fact, in the context of a repeated prisoners' dilemma game,

[3]In particular, Collins and Sherstyuk (2000) experimentally study a three firm location model in the unit interval with fixed prices and attribute the divergence between the theoretical predictions (Shaked 1982) –location of a risk neutral firm is a random draw from a uniform distribution with support $[\frac{1}{4}, \frac{3}{4}]$– and experimental evidence –location choices result in a bimodal distribution– to the fact that agents are risk-averse. They computationally get some approximate equilibria for a case of risk-averse agents and show that these approximate equilibrium predictions fit the data better than the Nash equilibrium of the risk-neutral scenario.

Sabater-Grande and Georgantzís (2002) have also reported a positive relation between risk aversion and competitive behavior. Whether this effect is due to a more general pattern linking risk aversion to competitive behavior or it relates to the specific underlying behavioral driving forces of a particular context needs certainly further investigation. Second, the external validity of risk elicitation methods based on lottery choices offered some further favorable evidence. Finally, the use of repeated choices under random matching seems to be a reliable experimental context for testing the predictions of theoretical mixed strategy equilibria, even in more complex, bi-dimensional decision-making contexts.

The remainder of the article is organized as follows: We present the theoretical model in section 2 and its formal results in section 3. Also in section 3, we briefly present an equilibrium of the original location-then-pricing game for comparative purposes. The experimental design and results are detailed in section 4 while in section 5 we provide some concluding remarks.

2 The model

We analyze a two-firm competition model in which firms simultaneously decide a product characteristic (location, l_i) and a price (p_i). Formally, each firm $i \in \{A, B\}$ chooses a catalog $c_i = (l_i, p_i) \in \{W, C, E\} \times [0, 1]$ where $\{W, C, E\} \subset \mathbb{R}$ is our discrete linear city (see Figure 1, W stands for the western district, C stands for the central district and E for the eastern district). For analytical tractability, we assume that $W = -E$, $C = 0$ and $E > 1$.

Insert Figure 1 about here

We also assume that there exists a unit mass of consumers whose residences are uniformly distributed on the linear city. Formally, we consider that a consumer $j \in [0, 1]$ resides at $h(j) \in \{W, C, E\}$ and, without loss of generality that $h(j) \leq h(j')$ for every $j < j'$. Each consumer buys exactly one unit of good from only one of the two firms. Considering that the utility of a consumer $j \in [0, 1]$ with a residence at $h(j) \in \{W, C, E\}$ from a certain catalog $c = (l, p)$ is given by

$$U_j(c) = -p - |l - h(j)|$$

we assume that this consumer buys the unit of good from firm A if $U_j(c_A) > U_j(c_B)$, from firm B if $U_j(c_A) < U_j(c_B)$ and with probability $\frac{1}{2}$ from each of the two firms if $U_j(c_A) = U_j(c_B)$. We moreover define the sets of consumers who strictly prefer each of the two catalogs as

$$I_A(c_A, c_B) = \{j \in [0,1] | U_j(c_A) > U_j(c_B)\}$$

$$I_B(c_A, c_B) = \{j \in [0,1] | U_j(c_A) < U_j(c_B)\}.$$

Then, the profits of the two firms as functions of their catalogs are given by

$$\Pi_A(c_A, c_B) = p_A \times [\mu(I_A(c_A, c_B)) + \tfrac{1-\mu(I_A(c_A,c_B))-\mu(I_B(c_A,c_B))}{2}]$$

$$\Pi_B(c_A, c_B) = p_B \times [\mu(I_B(c_A, c_B)) + \tfrac{1-\mu(I_A(c_A,c_B))-\mu(I_B(c_A,c_B))}{2}]$$

where $\mu(S)$ is the Lebesgue measure of the set $S \subset \mathbb{R}$. Hence, like Osborne and Pitchik (1987) and many other relevant models we assume zero marginal costs of production.

We consider that each firm $i \in \{A, B\}$ maximizes $v(\Pi_i(c_A, c_B))$ where $v : \mathbb{R} \to \mathbb{R}$ is any strictly increasing and absolutely continuous function; without loss of generality we normalize $v(0) = 0$. This very general structure of firm's preferences allows us to characterize an equilibrium for any kind of firm's risk preferences.

A mixed strategy profile in this set up is denoted by (σ_A, σ_B) where for each $i \in \{A, B\}$, $\sigma_i = (F_i^W(p), F_i^C(p), F_i^E(p))$. For each $i \in \{A, B\}$ and each $l \in \{W, C, E\}$, $F_i^l(p)$ is the probability that the catalog $c_i = (l_i, p_i)$ of firm $i \in \{A, B\}$ is such that $l_i = l$ and $p_i \leq p$. A Nash equilibrium in mixed strategies is a mixed strategy profile $(\hat{\sigma}_A, \hat{\sigma}_B)$ such that $\hat{\sigma}_B$ $(\hat{\sigma}_A)$ is a best response of firm B (A) to firm A (B) playing $\hat{\sigma}_A$ $(\hat{\sigma}_B)$.

Before we advance to the characterization results some comments regarding the employed assumptions are in order. The fact that $p_i \in [0, 1]$ simply captures that a reservation price exists or a government imposed threshold on the product's price in the spirit of Osborne and Pitchik (1987); obviously, the selection of a reservation price equal to one is without loss of generality. On the other hand the assumption $E > 1$ has certain implications on the analysis which is about to follow. This assumption essentially mitigates the intensity of competition between two firms who locate to distinct districts without eliminating it, as did the assumptions of earlier models of simultaneous catalog competition.[4] Our approach suggests that if the two firms locate at different districts then each of them will get the consumers of her district independently of the price choices that the competitors make. But this does not eliminate incentives to compete. If one firm locates at W and the other at E then the consumers who are located at C will buy the product from the firm which offers the lowest price: price competition has a significant intensity. That is, our model may be indeed a rough approximation of a general catalog competition model with a large product characteristics' space but, to our knowledge, it is the first one that captures the competition dynamics that are generated by the simultaneous choice of location and price. Since local-monopolies equilibria are not possible in our setting, the equilibrium outcomes should provide novel insights regarding the nature of oligopolistic competition. If one assumes that $E < 1$ then, indeed, price competition would be even more intense. It is very important to note here, though, that our *equilibrium is robust to considering values of* $E < 1$ *as long as they are not very small*. Straightforwardly, if we let E take any arbitrarily small value then: a) the intensity of price competition will lead equilibrium prices near zero, b) derivation of complete analytical results would become intractable; and c) the dynamics which form the qualitative features of our equilibrium would not alter in any substantial way. Hence, we work with this assumption acknowledging its limitations but, at the same time, insisting that it permits that the model's dynamics unfold effectively.

[4] For instance, Economides (1984, 1987) considers that consumers have a very low valuation of the good –only the consumers located very near a "shop" will buy the good– and, hence, pure strategy equilibria in the catalog game exist such that each firm is a local monopolist. In our case this can never occur as price competition is intense at a non-degenerate degree.

3 Formal Analysis

3.1 Main Results

The game clearly does not admit any pure strategy equilibrium. If an equilibrium existed in which both firms charged the same positive price, then this should be a minimum differentiation equilibrium (since in all other cases each firm would have incentives to approach the other and marginally undercut the price). But if in a minimum differentiation equilibrium firms charge the same price then each firm has incentives to marginally undercut the other and double her profit. If in equilibrium firms charge different prices then the low price firm has incentives to locate at the same location as the high price firm and take all the market, and the high price firm has incentives to go far away from the low price firm –such an equilibrium cannot exist. Finally, it cannot be the case that both firms choose in equilibrium prices equal to zero. In such a case one of these firms could get positive profits by moving away from the other firm and by charging a positive price.

In order to improve the way our formal results are presented let us first give a couple of useful definitions.

Definition 1 *We say that* $(\hat{\sigma}_A, \hat{\sigma}_B)$ *is a symmetric equilibrium if (i)* $\hat{\sigma}_A = \hat{\sigma}_B$ *and (ii)* $\hat{F}_A^W(p) = \hat{F}_A^E(p)$ *for every* $p \in [0, 1]$.

Symmetry in our analysis means both that the two firms employ the same strategy –the standard game-theoretic meaning of the term– and that each firm uses a mixed strategy which is symmetric about the center of the linear city. This notion of symmetry is used in unidimensional spatial models with mixed equilibria (see, for example, Aragonès and Palfrey 2002).

Definition 2 *Consider that* $(\hat{\sigma}_A, \hat{\sigma}_B)$ *is a symmetric equilibrium. Then,* $q = \hat{F}_A^W(1)$, $G(p) = \frac{\hat{F}_A^W(p)}{\hat{F}_A^W(1)}$ *and* $Z(p) = \frac{\hat{F}_A^C(p)}{\hat{F}_A^C(1)}$

A mixed strategy $\sigma_i = (F_i^W(p), F_i^C(p), F_i^E(p))$ in our setup is bidimensional. The above definition simply defines conditional probability distributions which will help us present our result in a more intuitive manner. First, q is the probability that a firm locates at the western (eastern) district; obviously, $1-2q$ is the probability that a firm locates at the central district. Then $G(p)$ is the probability distribution of the price of a firm conditional on that this firm locates in the western (eastern) district. Finally, $Z(p)$ is the probability distribution of the price of a firm conditional on that this firm locates in the central district.

We can now state our main finding.

Proposition 1 *There exists a unique symmetric equilibrium* $(\hat{\sigma}_A, \hat{\sigma}_B)$ *which is such that*

$$\textit{(i)}\ \hat{F}_A^W(p) = \begin{cases} 0 & \text{if } p \in [0, p') \\ \frac{v(\frac{1}{3})(1-\frac{2v(\frac{2}{3})(v(\frac{1}{3})-v(\frac{p}{3}))}{v(\frac{1}{3})(-v(\frac{p}{3})+v(\frac{2p}{3})+v(p))})}{v(\frac{1}{3})+2v(\frac{2}{3})} & \text{if } p \in [p', 1] \\ \frac{v(\frac{1}{3})}{v(\frac{1}{3})+2v(\frac{2}{3})} & \text{if } p > 1 \end{cases}$$

$$\textit{(ii)}\ \hat{F}_A^C(p) = \begin{cases} 0 & \text{if } p \in [0, p'') \\ \frac{2v(\frac{1}{3})\left(v(\frac{2p}{3})-v(\frac{2}{3})\right)-\left(v(\frac{1}{3})-2v(\frac{2}{3})\right)v(p)}{\left(v(\frac{1}{3})+2v(\frac{2}{3})\right)v(p)} & \text{if } p \in [p'', 1] \\ 1-2\frac{v(\frac{1}{3})}{v(\frac{1}{3})+2v(\frac{2}{3})} & \text{if } p > 1 \end{cases}$$

$$\textit{(iii)}\ 0 < p'' < p' < 1$$

where $p' \in (0,1)$ *is the unique solution of* $v(\frac{1}{3})(-v(\frac{p'}{3}) + v(\frac{2p'}{3}) + v(p')) = 2v(\frac{2}{3})(v(\frac{1}{3}) - v(\frac{p'}{3}))$ *and* $p'' \in (0,1)$ *is the unique solution of* $2v(\frac{1}{3})\left(v(\frac{2p''}{3}) - v(\frac{2}{3})\right) = \left(v(\frac{1}{3}) - 2v(\frac{2}{3})\right) v(p'')$.

Proof. Assume that a symmetric equilibrium $(\hat{\sigma}_A, \hat{\sigma}_B)$ exists and denote by $S \subseteq \{W, C, E\} \times [0,1]$ the support of its underlying probability distribution. We will first show that: a) $(\hat{\sigma}_A, \hat{\sigma}_B)$ is atomless and that b) S has no gaps.[5] By S having no gaps we mean that if for some $l \in \{W, C, E\}$ two distinct catalogs $(l, \dot{p})$ and $(l, \ddot{p})$ belong to S then $F_A^l(\dot{p}) \neq F_A^l(\ddot{p})$. The reason why any symmetric equilibrium is atomless is straightforward. Assume the contrary, that is, that there exists a mass point on catalog $(W, \dot{p}) \in S$: firm B expects that firm A will choose catalog $(W, \dot{p}) \in S$ with probability

[5] Establishing first that the equilibrium involves no mass points and no gaps is known to greatly simplify the equilibrium characterization (see, for example, Burdett and Judd 1983; and Gabszewicz et al. 2008).

$F_A^W(\dot{p}) - \lim_{p \to \dot{p}^-} F_A^W(p) > 0$. First of all, it is trivial to see why this can never be the case for a catalog $(W, \dot{p})$ with $\dot{p} = 0$. If such a catalog is part of S then, in this symmetric equilibrium, the expected payoff of each firm is zero. But in this game a player can always secure positive expected payoffs if, for example, she mixes uniformly among $(W, 1)$, $(C, 1)$ and $(E, 1)$. Therefore if there exists a mass point at $(W, \dot{p})$ it must be such that $\dot{p} > 0$. If firm B chooses $(W, \dot{p}) \in S$ her payoff will be

$$[F_A^W(\dot{p}) - \lim_{p \to \dot{p}^-} F_A^W(p)]v(\tfrac{\dot{p}}{2}) + [F_A^W(1) - F_A^W(\dot{p})]v(\dot{p}) + F_A^C(1)v(\tfrac{\dot{p}}{3}) +$$

$$+[F_A^W(\dot{p}) - \lim_{p \to \dot{p}^-} F_A^W(p)]v(\tfrac{\dot{p}}{2}) + [F_A^W(1) - F_A^W(\dot{p})]v(\tfrac{2\dot{p}}{3}) + [\lim_{p \to \dot{p}^-} F_A^W(p)]v(\tfrac{\dot{p}}{3})$$

and if B chooses $(W, \dot{p} - \varepsilon)$ her payoff will be

$$[F_A^W(\dot{p} - \varepsilon) - \lim_{p \to \dot{p} - \varepsilon^-} F_A^W(p)]v(\tfrac{\dot{p} - \varepsilon}{2}) + [F_A^W(1) - F_A^W(\dot{p} - \varepsilon)]v(\dot{p} - \varepsilon) + F_A^C(1)v(\tfrac{\dot{p} - \varepsilon}{3}) +$$

$$+[F_A^W(\dot{p} - \varepsilon) - \lim_{p \to \dot{p} - \varepsilon^-} F_A^W(p)]v(\tfrac{\dot{p} - \varepsilon}{2}) + [F_A^W(1) - F_A^W(\dot{p} - \varepsilon)]v(\tfrac{2(\dot{p} - \varepsilon)}{3}) + [\lim_{p \to \dot{p} - \varepsilon^-} F_A^W(p)]v(\tfrac{\dot{p}}{3}).$$

We can obviously find $\varepsilon > 0$ arbitrary small such that the considered mixed strategy has no mass at $(W, \dot{p} - \varepsilon)$. Therefore, when B chooses $(W, \dot{p} - \varepsilon)$ and $\varepsilon \to 0$ her expected payoff becomes

$$[F_A^W(1) - \lim_{p \to \dot{p}^-} F_A^W(p)]v(\dot{p}) + F_A^C(1)v(\tfrac{\dot{p}}{3}) + [F_A^W(1) - \lim_{p \to \dot{p}^-} F_A^W(p)]v(\tfrac{2\dot{p}}{3}) + [\lim_{p \to \dot{p} - \varepsilon^-} F_A^W(p)]v(\tfrac{\dot{p}}{3})$$

which is strictly larger than her payoff at $(W, \dot{p})$. That is, $(W, \dot{p})$ cannot belong to the support of a mixed strategy which characterizes a symmetric equilibrium because firms prefer other catalogs to that. This suggests that our assumption –that the symmetric equilibrium might be such that there exists a mass point on a certain catalog $(W, \dot{p})$– is wrong. An argument which rules out existence of a mass point at a catalog $(E, \dot{p})$ is symmetric to the one that we just developed here, and, an argument which rules out existence of a mass point at a catalog $(C, \dot{p})$ is also very similar to the present one. This concludes the proof that if a symmetric equilibrium exists then the underlying probability distribution has no atoms.

We now turn attention to our second claim: if a symmetric equilibrium exists then it has no gaps. Again assume that the contrary is true. Consider that we have a gap and, hence, there exist two distinct catalogs $(W, \dot{p})$ and $(W, \ddot{p})$ which belong to S and which are such that $\dot{p} < \ddot{p}$ and $F_A^W(\dot{p}) =$

$F_A^W(\ddot{p})$. Then, given that this symmetric equilibrium is atomless, if B chooses $(W, \dot{p})$ her payoff will be $[F_A^W(1) - F_A^W(\dot{p})]v(\dot{p}) + F_A^C(1)v(\frac{\dot{p}}{3}) + [F_A^W(1) - F_A^W(\dot{p})]v(\frac{2\dot{p}}{3}) + F_A^W(\dot{p})v(\frac{\dot{p}}{3})$,

while if she chooses $(W, \ddot{p})$ her payoff will be

$$[F_A^W(1) - F_A^W(\ddot{p})]v(\ddot{p}) + F_A^C(1)v(\tfrac{\ddot{p}}{3}) + [F_A^W(1) - F_A^W(\ddot{p})]v(\tfrac{2\ddot{p}}{3}) + F_A^W(\ddot{p})v(\tfrac{\ddot{p}}{3}).$$

Since $F_A^W(\dot{p}) = F_A^W(\ddot{p})$ it trivially follows that B's payoff at $(W, \ddot{p})$ is strictly larger than her payoff at $(W, \dot{p})$. Therefore, $(W, \dot{p})$ cannot be part of the support of a symmetric equilibrium mixed strategy. Arguments which rule out existence of gaps in $F_A^C(p)$ and in $F_A^E(p)$ are very similar. Hence, if a symmetric equilibrium exists it must have no gaps.

Knowing that if a symmetric equilibrium exists it is an atomless equilibrium with no gaps is very useful for our characterization attempt. Notice that the above arguments moreover establish that $(W, 1)$, $(C, 1)$ and $(E, 1)$ are all part of S. Hence, in a symmetric equilibrium the expected payoff, v^*, of each of the firms should coincide with the payoff of firm B when firm A is expected to play the equilibrium mixed strategy and firm B to choose $(W, 1)$ or to choose $(C, 1)$. In the first case (when B plays $(W, 1)$) the equilibrium payoff, v^*, can be shown to be equal to $F_A^C(1)v(\frac{1}{3}) + F_A^W(1)v(\frac{1}{3}) = [1 - F_A^W(1)]v(\frac{1}{3})$. And in the second case (when B plays $(C, 1)$), the equilibrium payoff, v^*, can be shown to be equal to $2F_A^W(1)v(\frac{2}{3})$.

Therefore, in a symmetric equilibrium we must have

$$F_A^W(1) = \frac{v(\frac{1}{3})}{v(\frac{1}{3})+2v(\frac{2}{3})},\ F_A^C(1) = 1 - 2\frac{v(\frac{1}{3})}{v(\frac{1}{3})+2v(\frac{2}{3})} \text{ and } v^* = \frac{2v(\frac{1}{3})v(\frac{2}{3})}{v(\frac{1}{3})+2v(\frac{2}{3})}.$$

Since S has no gaps and since there are no mass points involved, if firm B chooses $(W, p) \in S$ it must be the case that

$$[\frac{v(\frac{1}{3})}{v(\frac{1}{3})+2v(\frac{2}{3})} - F_A^W(p)]v(p) + [1-2\frac{v(\frac{1}{3})}{v(\frac{1}{3})+2v(\frac{2}{3})}]v(\tfrac{p}{3}) + [\frac{v(\frac{1}{3})}{v(\frac{1}{3})+2v(\frac{2}{3})} - F_A^W(p)]v(\tfrac{2p}{3}) + F_A^W(p)v(\tfrac{p}{3}) = \frac{2v(\frac{1}{3})v(\frac{2}{3})}{v(\frac{1}{3})+2v(\frac{2}{3})}$$

which is equivalent to $F_A^W(p) = \frac{v(\frac{1}{3})(1-\frac{2v(\frac{2}{3})(v(\frac{1}{3})-v(\frac{p}{3}))}{v(\frac{1}{3})(-v(\frac{p}{3})+v(\frac{2p}{3})+v(p))})}{v(\frac{1}{3})+2v(\frac{2}{3})}$.

We notice that $F_A^W(p) \geq 0$ if and only if $p \geq p'$ where $p' \in (0, 1)$ is the unique solution of $v(\frac{1}{3})(-v(\frac{p'}{3}) + v(\frac{2p'}{3}) + v(p')) = 2v(\frac{2}{3})(v(\frac{1}{3}) - v(\frac{p'}{3}))$.

Moreover, if firm B chooses $(C,p)\in S$ it must be the case that

$$2\frac{v(\frac{1}{3})}{v(\frac{1}{3})+2v(\frac{2}{3})}v(\frac{2p}{3})+[(1-2\frac{v(\frac{1}{3})}{v(\frac{1}{3})+2v(\frac{2}{3})})-F_A^C(p)]v(p)=\frac{2v(\frac{1}{3})v(\frac{2}{3})}{v(\frac{1}{3})+2v(\frac{2}{3})}$$

which is equivalent to $F_A^C(p)=\frac{2v(\frac{1}{3})\left(v(\frac{2p}{3})-v(\frac{2}{3})\right)-\left(v(\frac{1}{3})-2v(\frac{2}{3})\right)v(p)}{\left(v(\frac{1}{3})+2v(\frac{2}{3})\right)v(p)}$.

We notice that $F_A^C(p)\geq 0$ if and only if $p\geq p''$ where $p''\in(0,1)$ is the unique solution of $2v(\frac{1}{3})\left(v(\frac{2p''}{3})-v(\frac{2}{3})\right)=\left(v(\frac{1}{3})-2v(\frac{2}{3})\right)v(p'')$.

So, there exists a unique candidate for a symmetric equilibrium given by $(\hat{\sigma}_A,\hat{\sigma}_B)$, the strategy described in the statement of Proposition 1.[6]

To verify that indeed this symmetric strategy profile is a Nash equilibrium we trivially compute the expected payoff of a firm which chooses (l,p) when the other firm is expected to play according to the specified mixed strategy and we get that if (l,p) belongs to the support of the specified mixed strategy the firms expected payoff is $\frac{2v(\frac{1}{3})v(\frac{2}{3})}{v(\frac{1}{3})+2v(\frac{2}{3})}$ and if (l,p) does not belong to the support of the specified mixed strategy the expected payoff of the firm is strictly smaller than $\frac{2v(\frac{1}{3})v(\frac{2}{3})}{v(\frac{1}{3})+2v(\frac{2}{3})}$. Hence, a best response of a firm which expects that her competitor will behave according to the specified mixed strategy is to employ the same mixed strategy. This concludes the proof. ■

Notice that in this unique symmetric equilibrium we have $q<\frac{1}{3}$ and, hence, a firm locates to the central district with a probability strictly larger than $\frac{1}{3}$ and it locates to each of the peripheral districts with a probability strictly smaller than $\frac{1}{3}$. This is a very strong result as it holds *for any risk attitude* on behalf of the firms.

To better understand the nature of this unique symmetric equilibrium let us use a specific functional form of firms' risk preferences and assume that

[6] To see why $0<p''<p'<1$ holds, assume on the contrary that $p''\geq p'>0$. Then, if a firm chooses (W,p'), it expects a payoff equal to $qv(p')+(1-2q)v(\frac{p'}{3})+qv(\frac{2p'}{3})$ and if a firm chooses (C,p'), it expects a payoff equal to $(1-2q)v(p')+2qv(\frac{2p'}{3})$, with $q=\frac{v(\frac{1}{3})}{v(\frac{1}{3})+2v(\frac{2}{3})}$ and $(1-2q)v(p')+2qv(\frac{2p'}{3})\leq qv(p')+(1-2q)v(\frac{p'}{3})+qv(\frac{2p'}{3})$. Notice that the last inequality reduces to $v(\frac{2p'}{3})\leq\frac{(1-2q)v(\frac{p'}{3})+(3q-1)v(p')}{q}$, and since $q<\frac{1}{3}$ for any admissible v, it follows that $1-2q>0$ and $3q-1<0$; and subsequently that $\frac{(1-2q)v(\frac{p'}{3})+(3q-1)v(p')}{q}<\frac{(1-2q)v(\frac{p'}{3})+(3q-1)v(\frac{p'}{3})}{q}=v(\frac{p'}{3})$. That is, $v(\frac{2p'}{3})\leq v(\frac{p'}{3})$, which is impossible for any admissible v.

$$v(x) = \begin{cases} \frac{1-e^{-ax}}{1-e^{-a}} & \text{if } a \neq 0 \\ x & \text{if } a = 0 \end{cases}.$$

That is $v(x)$ exhibits constant absolute risk aversion (CARA), its risk aversion parameter is $a \in \mathbb{R}$ (if $a > 0$ the firm is risk-averse, if $a = 0$ the firm is risk-neutral and if $a < 0$ the firm is risk-loving), it is continuous for every $x \in [0,1]$ and every $a \in \mathbb{R}$; and $v(0) = 1$ and $v(1) = 1$ for every $a \in \mathbb{R}$.

Given the above, our unique symmetric equilibrium $(\hat{\sigma}_A, \hat{\sigma}_B)$ becomes such that

$$\hat{F}_A^W(p) = \begin{cases} 0 & \text{if } p \in [0, p') \\ -\frac{e^{-a/3}\left(e^{2a/3}-2e^{ap}-2e^{a\left(\frac{1}{3}+p\right)}-e^{a\left(\frac{2}{3}+p\right)}+e^{\frac{2}{3}a(1+p)}+e^{\frac{1}{3}a(2+p)}+2e^{\frac{1}{3}(a+2ap)}\right)}{\left(2+3e^{a/3}\right)\left(-1+e^{\frac{ap}{3}}\right)\left(1+e^{\frac{ap}{3}}\right)^2} & \text{if } p \in [p', 1] \\ \frac{1}{3+2e^{-a/3}} & \text{if } p > 1 \end{cases}$$

and

$$\hat{F}_A^C(p) = \begin{cases} 0 & \text{if } p \in [0, p'') \\ \frac{e^{-a/3}\left(-2e^{a/3}-e^{2a/3}+2e^{ap}+2e^{a\left(\frac{1}{3}+p\right)}+e^{a\left(\frac{2}{3}+p\right)}-2e^{\frac{1}{3}a(2+p)}\right)}{\left(2+3e^{a/3}\right)(-1+e^{ap})} & \text{if } p \in [p'', 1] \\ 1 - \frac{2}{3+2e^{-a/3}} & \text{if } p > 1 \end{cases}$$

for $0 < p'' < p' < 1$ which satisfy $\hat{F}_A^W(p') = 0$ and $\hat{F}_A^C(p'') = 0$.

In particular for the case of risk neutral firms, that is, for $v(x) = x$, our equilibrium significantly simplifies to

$$\hat{F}_A^W(p) = \begin{cases} 0 & \text{if } p \in [0, \frac{1}{2}) \\ \frac{2}{5} - \frac{1}{5p} & \text{if } p \in [\frac{1}{2}, 1] \\ \frac{1}{5} & \text{if } p > 1 \end{cases}$$

and

$$\hat{F}_A^C(p) = \begin{cases} 0 & \text{if } p \in [0, \frac{4}{13}) \\ \frac{13}{15} - \frac{4}{15p} & \text{if } p \in [\frac{4}{13}, 1] \\ \frac{3}{5} & \text{if } p > 1 \end{cases} .$$

In Table 1 we present some brief information about our equilibrium for various values of the risk parameter $a \in \mathbb{R}$.

Insert Table 1 about here

Notice that $q < \frac{1}{4}$ when the players are risk-loving or even mildly risk-averse. That is, *firms produce more often ($1 - 2q > \frac{1}{2}$) mainstream products than specialized ones, unless they are substantially risk-averse.* Moreover, in Figure 2, we present the evolution of cumulative distribution functions of the conditional price distributions, $G(p)$ and $Z(p)$, for various risk aversion levels.

Insert Figure 2 about here

3.2 First-location-then-pricing

In order to study the effect of sequential location and pricing on the outcomes, we also characterize an equilibrium in the first-location-then-pricing game. When studying such a version of the game, we need to notice that price choices of a firm when she decides to locate at W, for example, crucially depend on the location choice of her competitor: if the competitor also locates at W then the pricing game is very tough as the firm who sets the smaller price gets all the market, if the competitor locates at E then each firm serves for sure the population located at her position and competes for the central market share, and when the competitor locates in the centre, then the competitor serves the population at C and E and the firm located at W serves the population located at her position without particular tensions. That is, when firms locate: a) at the same position, then there is *maximum competitiveness* in the pricing game, b) at different extremes, then there is *intermediate competitiveness* and c) asymmetrically –one at C and the other either at W or at E–, then there is *minimum competitiveness*.[7]

In the catalog competition game studied above, indeed, it made sense to condition pricing strategies on given own location choices. As it is evident from the presented arguments –and from the formal analysis of the model– in the sequential version of the game one should study pricing strategies not conditional on own location choices, but on the realized pairs of locations and hence on the degree of pricing competitiveness that these location choices induce.

To produce a prediction that would be comparable to the equilibrium of the catalog competition game we consider, as Bester et al. (1996), that there is no coordination device in the location selection stage, and hence we look for equilibria such that players mix symmetrically among the available locations.

Proposition 2 *In the first-location-then-price version of the game there exists a unique symmetric subgame perfect equilibrium which is such that (i) in the location stage each firm locates to the western (eastern) district with a probability equal to* $\phi = \frac{v(\frac{1}{3})}{v(\frac{1}{3})+2v(\frac{2}{3})}$ *and to the central district with probability* $1 - 2\phi = \frac{2v(\frac{2}{3})-v(\frac{1}{3})}{v(\frac{1}{3})+2v(\frac{2}{3})}$ *and (ii) if firms locate at the same district (maximum competitiveness) they both set prices equal to zero, if one firm locates at the western district and one at the eastern district (intermediate competitiveness) then each of them draws its price from* $\Psi(p) = \frac{-v(\frac{1}{3})+v(\frac{2p}{3})}{-v(\frac{p}{3})+v(\frac{2p}{3})}$ *for* $p \in [\frac{1}{2}, 1]$*, and if firms locate at neighboring districts (minimum competitiveness) they both set prices equal to one.*[8]

This proposition shows that the probability that a firm locates at each of the three districts is identical to the probability of the unique symmetric equilibrium of the catalog competition game. In the framework of the specific CARA function that we introduced above it is not difficult to see that for $a \in [0 + \infty)$ we have that $\phi \in [\frac{1}{5}, \frac{1}{3})$ and therefore the probability that the two firms employ in the second stage of the game a strategy profile with prices equal to one or zero is always significantly large. As far as pricing is concerned, it mostly depends on the realized degree of competitiveness. Naturally, risk aversion affects the equilibrium pricing strategy only when competitiveness is intermediate, as in

[7]For an analysis of factors that affect price competition dynamics beyond product similarity one is referred to Cabral and Villas-Boas (2005).

[8]This proposition is derived using standard arguments and, thus, its proof is omitted.

the other two cases the pricing equilibrium is in pure strategies and hence not affected by the players' risk attitudes. In Figure 3 we present pricing CDFs conditional on the exact degree of competitiveness for various risk attitudes.

Insert Figure 3 about here

When a converges to minus infinity, that is when firms become extremely risk-loving, then both in the catalog and in the two-stage game the probability that both of them locate at the center converges to one, but in the catalog version of the game the price that they will charge is, with a probability that converges to one, arbitrarily close to $\frac{2}{3}$ while in the two-stage game, if they both locate in the central district they both charge zero. That is, when the competing firms are extremely risk-loving both models produce minimum differentiation outcomes with the difference that prices under catalog competition are very large (around $\frac{2}{3}$) while in the standard two-stage game they converge to zero. What we observe in Figures 2 and 3 is that, independently of the degree of risk aversion, the two-stage game delivers less smooth and more extreme pricing since pricing choices depend on realized locations of both players.

4 Experiment

4.1 Design

An experiment was conducted to test the predictions of the theory, focusing on the role of risk aversion and on the effect of sequential (as opposed to simultaneous) location and pricing decisions. A total of 120 students (gender balanced) were recruited, following the standard procedures at LINEEX, University of Valencia (Spain). The experiment was programmed in z-Tree (Fischbacher 2007). Each experimental session consisted of two consecutive sub-sessions. Subjects received their rewards at the end of the second sub-session as explained below.

In the first sub-session, we elicited subjects' risk preferences using the lottery panel test introduced by Sabater-Grande and Georgantzís (2002). As discussed elsewhere[9], subjects' choices in this test can be used as a reliable source of information on risk attitudes as an explanatory variable of behavior in other economic decision making contexts. Furthermore, while the test offers a multidimensional characterization of risk attitudes, we focus on risk aversion alone, in order to remain as close as possible to the theoretical model presented in the previous section. A detailed description of the test can be found in the Appendix.

In the second sub-session, subjects were randomly allocated to one of two experimental treatments, SIM (catalog competition) or SEQ (first-location-then-pricing). The hypotheses of the catalog competition model are tested using data from the SIM treatment alone, while SEQ data are used to empirically compare simultaneous location and pricing to the standard first-location-then-pricing framework. A single 60-subject session lasting 25 periods was run under each treatment. In each session, subjects where matched within 6 independent 10-subject groups in order to form, in each period, random pairs of firms interacting according to the rules of the corresponding simultaneous or sequential location and pricing game. Written instructions were given to them, which were also read aloud by one of the organizers.[10] Subjects' understanding of the instructions was tested through a pre-play questionnaire.

Average per subject earnings were 5 and 13 Euros from each sub-session respectively. The total duration of a session was approximately 90 minutes (less than 20 minutes corresponded to the first sub-session, including instructions, and the remaining corresponded to the second sub-session, including payment).

4.2 Hypotheses

Using the experimental data, the following hypotheses emerging from the theoretical framework can be tested:

H1: Central locations will be chosen more frequently than extreme locations both in SIM and in SEQ.

[9]See, for example, Sabater-Grande and Georgantzís (2002), Georgantzís and Navarro-Martínez (2010) and especially, García-Gallego et al. (2012).

[10]An English translation of the instructions in Spanish is provided in the Appendix.

H2: Prices will be higher for extreme location choices than for central ones in SIM.

H3: Prices will be decreasing in the competitiveness of the realized locations in SEQ.

H4: Given a location choice, a subject's degree of risk aversion will have a negative impact on the price level.

Hypothesis 1 directly follows from the fact that in our formal results regarding both SIM and SEQ, the single most frequent choice is the central location for any admissible utility function (see Propositions 1 and 2 and their proofs); Hypothesis 2 is due to the fact that $p'' < p'$ (see Proposition 1); Hypothesis 3 follows from the equilibrium relationship between competitiveness and price level (see Proposition 2); and Hypothesis 4 is a consequence of the parametric study of the equilibrium properties using the CARA utility functions (see the discussion that follows Propositions 1 and 2).

4.3 Results

We first present and discuss descriptive statistics (in Table 2) and graphics and then we proceed with formal econometric analysis.

Insert Table 2 about here

Figure 4 presents shares of location choices by individual participant in treatment SIM. Most of the subjects have chosen both extreme and central locations, which is compatible with the use of mixed strategies. Only six (out of 60) subjects (subject 3, 12, 18, 22, 31 and 35) have chosen the same type of location (central or extreme) throughout the session. Of them, only two subjects (3 and 31) have always located on an extreme.[11]

Insert Figure 4 about here

[11] We refer to each subject by his unique identification number in the database we have generated. Numbers from 1 to 60 are the id for subjects in SIM, while 61 to 120 are the id numbers for subjects in SEQ.

In fact, compatible with the theoretical prediction, subjects in SIM have chosen central locations more frequently (about 55%) than extreme ones (see Figure 5). These observations offer support to hypothesis *H1*. Also, note that observed locations are compatible with moderate risk aversion (see the case of a=1 in Table 1).

Insert Figure 5 about here.

As can be seen in Figure 6 –which nicely relates to theoretical predictions of Figure 2– and in Figure 7, prices have been significantly lower at central locations than at extreme ones, with more than half of them being equal to the consumers' reservation price when a firm is located on an extreme. This offers support to *H2*.

Insert Figure 6 about here

Insert Figure 7 about here

Finally, Figure 8 offers some support for *H4*, regarding the negative effect of subject's risk aversion on pricing.

Insert Figure 8 about here

However, this and other findings from the experiments are formally addressed using the econometric model whose estimation results and other diagnostic tests are reported in Table 2, that we comment later on.

We now focus on the comparison of the effects of simultaneous vs. sequential location and pricing based on the experimental data.

First of all, it is worth noting the similarities between behavior in SEQ and that reported for SIM. For example, Figure 9 indicates that, like in SIM, very few subjects, (only 6 out of 60, subjects 69, 72, 74, 97, 102 and 118) have chosen the same type of location (central or extreme) throughout the session. Of them, only subject 97 has always located on an extreme. All other subjects have chosen both extreme and central locations. Also, as shown in Figure 10, subjects have chosen central locations more frequently (approximately 64%) than extreme ones.

Insert Figure 9 about here

Insert Figure 10 about here

The effect of the degree of *competitiveness* is presented in Figure 11 –which, also, nicely relates to theoretical predictions of Figure 3– while aggregate competitiveness realizations are depicted on Figure 12. As explained, competitiveness is maximum when both firms locate at the same position, and minimum when one is at the center and the other at one extreme. Intermediate competitiveness corresponds to the case in which both firms symmetrically locate on the extremes of the segment. Using this definition, we observe that the profiles emerging from location choices are very similar across treatments.

Insert Figure 11 about here

Insert Figure 12 about here

Note that due to the different temporal structures of SIM and SEQ, we should account for the information available to subjects when deciding on prices. Thus, we use location choices (central vs. extreme) in SIM and full location profiles (minimum, intermediate and maximum competitiveness) in SEQ, because in the latter a firm choosing price has already observed its rival's location. Consistent with the predictions of the theoretical model, prices have been close to maximal for extreme locations but much lower in central ones in SIM. Furthermore, prices in SEQ follow the expected pattern, as can be observed in Figure 13, with almost global adoption of monopoly pricing for minimum levels of competitiveness and much lower prices for maximum competitiveness. Intermediate competitiveness leads to prices which are close to monopoly, but lower than under minimum competitiveness. These facts offer support to hypothesis *H3*.

Insert Figure 13 about here

Besides the descriptive graphical analysis we have also run an econometric model on the data to jointly test the effect of the different variables that our theoretical model presented as determinant for the pricing decision: the degrees of competitiveness and risk aversion. We have also added a lagged price variable, as Cason and Friedman (2003) already detected serial and cross sectional correlation in experimental pricing decisions. And also a period variable to account for learning. The particular modeling choice: correlated panels corrected standard errors is the more suited given the nature of our data, but a standard OLS regression with robust standard errors would offer virtually identical results.

As we compare the two treatments in the same regression, the coefficients obtained for the SEQ treatment express only the difference with those estimated for the SIM model (which acts as the baseline). That is, the value estimated for SEQ should be added to (or subtracted from if negative) that of SIM to obtain the coefficient corresponding to SEQ. The advantage of this way of presenting the results is that we directly know, by looking at the p-values for SEQ, whether the difference between the coefficients corresponding to the two treatments is significant or not. The only exception to this pattern is the variable Competitiveness, as we are forced to consider current competitiveness level in the sequential case and lagged competitiveness level in the simultaneous case (because the current one cannot be known by the subject at the time of the decision). For this particular variable the value estimated for SEQ is the coefficient and not the difference in coefficients as in all other variables.

Insert Table 3 about here

First of all, by looking at the constant, we observe that catalog competition in our experiment leads to significantly lower prices than in sequential location-then- pricing. In fact, the negative effect of competitiveness on price (H3) is much stronger in SEQ, given that in this treatment subjects observe the full present location profile rather than just the lagged one, which is the case in SIM. The picture is enriched by looking at the dynamics of price strategies. A subject's price in one period is higher, the higher the price has been in the previous period and the effect is stronger in SIM. At first sight, the correlation of individual prices across subsequent periods might seem to be at odds with the hypothesis of randomized strategies required in a mixed strategy equilibrium. However, the one-shot interaction within pairs of strangers makes it impossible for subjects to guess their rivals' prices in one period from prices they have observed in the past. The negative effect of risk aversion on prices (H4) is also confirmed by the econometric results. Experience with the game has driven prices up in both treatments.

5 Conclusions

In this paper, we have obtained the equilibrium in a simultaneous discrete location-pricing game of catalog competition. Given the lack of pure strategy equilibria, we have characterized the equilibrium paying special attention to the role of risk aversion, which gains importance, in light of its mixed-strategy nature. Indeed, subsequent studies should pay great attention in adding to this model a richer locations' space, general consumers' distributions and preferences (allowing, for instance, consumers to enjoy product variety in the spirit of Sajeesh and Raju 2010), in order to validate that the intuition that we got from the present analysis continues to hold in alternative settings. We have to stress though that the present game allows for non-trivial pricing-location interactions to properly develop, and hence our results are arguably not limited to the described specification.

We have experimentally tested the predictions of the model regarding both the role of simultaneity of location and pricing and the effect of risk aversion. A fundamental difference between the sequential and the simultaneous version of the model studied here, which is confirmed by our experimental results, is the prediction that, in catalog competition, firms never choose very low prices even when they produce the same product variety, unlike in the two-stage game in which firms end up setting their prices equal to their marginal cost when their goods are not differentiated. Experimentally, simultaneous and sequential competition are fundamentally different, with past prices and competitiveness of the locations taking a more salient role in the former and current competitiveness of the locations in the latter. However, other theoretical predictions common to both models are also confirmed by our experiment: central locations are more frequently chosen than extreme locations (compatible with moderate degrees of risk aversion), prices are lower when competitiveness is high, and, given a location profile, risk aversion is found to lead to lower prices. A natural next step would be to allow firms to determine the timing of their actions endogenously –catalog vs. first-location-then-pricing– and we defer such a more elaborate endeavour to future research.

References

[1] Arad A., Rubinstein A.: The 1120 Money Request Game: A level-k reasoning study. American Economic Review 102(7), 3561-3573 (2012)

[2] Aragonès E., Palfrey T.R.: Mixed strategy equilibrium in a Downsian model with a favored candidate. Journal of Economic Theory 103, 131-161 (2002)

[3] Aragonès E., Palfrey T.R.: The effect of candidate quality on electoral equilibrium: an experimental study. American Political Science Review 98(1), 77-90 (2004)

[4] Barreda-Tarrazona I., García-Gallego A., Georgantzís N., Andaluz-Funcia J., Gil-Sanz A.: An experiment on spatial competition with endogenous pricing. International Journal of Industrial Organization 29, 74-83 (2011)

[5] Baye M.R., Kovenock D., De Vries C.G.: The all-pay auction with complete information. Economic Theory 8(2), 291-305 (1996)

[6] Bester H., De Palma A., Leininger W., Thomas J., von Thadden E.L.: A noncooperative analysis of Hotelling's location game. Games and Economic Behavior 12(2), 165-186 (1996)

[7] Brown-Kruse J., Cronshaw M.B., Schenk D.J.: Theory and experiments on spatial competition. Economic Inquiry 31, 139-165 (1993)

[8] Brown-Kruse J., Schenk, D.J.: Location, cooperation and communication: An experimental examination. International Journal of Industrial Organization 18, 59-80 (2000)

[9] Burdett K., Judd K.L.: Equilibrium price dispersion. Econometrica 51(4), 955-969 (1983)

[10] Cabral L.M., Villas-Boas M.: Bertrand supertraps. Management Science 51(4), 599-613 (2005)

[11] Camacho-Cuena E., García-Gallego A., Georgantzís N., Sabater-Grande G.: Buyer-seller interaction in experimental spatial markets. Regional Science and Urban Economics 35, 89-105 (2005)

[12] Cason T., Friedman D.: Buyer search and price dispersion: A laboratory study. Journal of Economic Theory 112, 232-260 (2003)

[13] Collins R., Sherstyuk K.: Spatial competition with three firms: An experimental study. Economic Inquiry 38(1), 73-94 (2000)

[14] Dasgupta P., Maskin E.: The existence of equilibrium in discontinuous economic games, II: Applications. Review of Economic Studies 53, 27-41 (1986)

[15] Economides N.: The principle of minimum differentiation revisited. European Economic Review 24(3), 345-368 (1984)

[16] Economides N.: On Nash equilibrium existence and optimality in oligopolistic competition in prices and varieties. Greek Economic Review 9(2), 198-209 (1987)

[17] Fischbacher U.: zTree : Zurich toolbox for readymade economic experiments. Experimental Economics 10(2), 171-178 (2007)

[18] Fleckinger P., Lafay T.: Product flexibility and price competition in Hotelling's duopoly. Mathematical Social Sciences 60(1), 61-68 (2010)

[19] Gabszewicz J., Laussel D., Le Breton M.: The mixed strategy Nash equilibrium of the television news scheduling game. Journal of Industrial Economics 56, 418-425 (2008)

[20] García-Gallego A., Georgantzís N., Jaramillo-Gutiérrez A., Parravano M.: The lottery-panel task for bi-dimensional parameter-free elicitation of risk attitudes. Revista Internacional de Sociología 70(1), 53-72 (2012)

[21] Georgantzís N., Navarro-Martínez D.: Understanding the WTA-WTP gap: Attitudes, feelings, uncertainty and personality. Journal of Economic Psychology 31, 895-907 (2010)

[22] Hotelling H.: Stability in Competition. The Economic Journal 39, 41-57 (1929)

[23] Huck S., Müller W., Vriend N.: The East end, the West end, and King's cross: On clustering in the four-player Hotelling game. Economic Inquiry 40, 231-240 (2002)

[24] Hummel P.: On the nature of equilibriums in a Downsian model with candidate valence. Games and Economic Behavior 70(2), 425-445 (2010)

[25] Lerner A.P., Singer H.W.: Some notes on duopoly and spatial competition. Journal of Political Economy 45(2), 145-186 (1937)

[26] Monteiro P.K., Page Jr F.H.: Catalog competition and Nash equilibrium in nonlinear pricing games. Economic Theory 34(3), 503-524 (2008)

[27] Orzen H., Sefton M.: An experiment on spatial price competition. International Journal of Industrial Organization 26, 716-729 (2008)

[28] Osborne M.: Candidate positioning and entry in a political competition. Games and Economic Behavior 5, 133-151 (1993)

[29] Osborne M.J., Pitchik C.: Equilibrium in Hotelling's model of spatial competition. Econometrica 55(4), 911-922 (1987)

[30] Sabater-Grande G., Georgantzís N.: Accounting for risk aversion in repeated Prisoners' Dilemma Games: An experimental test. Journal of Economic Behavior and Organization 48, 37-51 (2002)

[31] Sajeesh S., Raju J.S.: Positioning and pricing in a variety seeking market. Management Science 56(6), 949-961 (2010)

[32] Shaked A.: Existence and computation of mixed strategy Nash Equilibrium for 3- firms location problem. Journal of Industrial Economics 31, 93-97 (1982)

6 Appendix

6.1 The lottery-panel elicitation task

The test is designed in the following way. Let a lottery (g, X) imply a probability q of earning X (else nothing). Consider a continuum of a such lotteries constructed to compensate riskier options with an increase in the expected payoff. Formally, such a continuum of lotteries will be defined by the pair (c, r) corresponding respectively, to the certain payoff c above which the expected payoff is increased by r times the probability of earning nothing. Therefore,

$$gX(g) = c + (1 - g) \cdot r \Rightarrow X(g) = \frac{c + (1 - g) \cdot r}{g}$$

In order to simplify the decision problem faced by our subjects, we use a discrete version (probabilities range between 1 and 0.1 in steps of 0.1) of the aforementioned setup, the *lottery panels* presented in Table A.1, which have been constructed using $c = 1$ and $r = 0.1, 1, 5, 10$ for panels 1 to 4 respectively. Subjects are asked to choose their preferred lottery from each panel, knowing that, at the end of the session, one of the panels will be randomly chosen to determine monetary payoffs and a number between 1 and 10 will be drawn to determine whether the favorable outcome emerges for the lottery chosen by each subject in this specific panel.

Insert Table A.1 about here

The higher the winning probability of the lottery chosen, the more risk averse the subject is. Risk neutral and risk loving subjects will choose the riskier option available to them.[12] Furthermore, under standard assumptions, expected utility maximizers would choose weakly riskier options as we move

[12]For example, a subject with utility $U(X) = X^{1/t}$ who is offered a panel of all possible lotteries designed under the linear risk-premium scheme described above would maximize his expected utility by choosing the lottery $(g^0, X(g^0))$, where $g^0 = (1 - \frac{1}{t}) \cdot (1 + \frac{c}{r})$.

The second order condition for a maximum is satisfied, as implied in $\frac{(1-t)\cdot(c+r)^2\cdot(\frac{c+(1-g)\cdot r}{g})^{1/t}}{g\cdot(c+(1-g)\cdot r)\cdot t^2} < 0$, as long as $t > 1$. This reflects the fact that risk-neutral ($t = 1$) and risk-loving ($t < 1$) subjects would choose the riskiest option available to them because their expected earnings are by design strictly decreasing in q.

from panel 1 to panel 4.[13]

However, while this is the expected pattern under standard uniparametric expected utility models, it is not necessarily true that all subjects monotonically choose lower probabilities as we move from Panel 1 to Panel 4. In fact, 25% of them usually violate this pattern. García-Gallego et al. (2012) show that the average choice across panels and the sensitivity of choices across panels are the two principal components describing a subject's behavior in this task. Given that using the second component would require either a biparametric expected utility model or a non expected utility framework, we use the first component alone, a subject's choice average, to describe subjects' risk aversion in a way compatible with our theoretical framework.

6.2 Instructions for treatment SIM (instructions specific to treatment SEQ in italics in parenthesis)

Welcome to this experiment and thank you for participating. The scope of this session is to study how individuals make decisions in specific contexts. The instructions are simple and, if you follow them carefully, you will receive privately an amount in cash at the end of the session. You may ask any question at any time raising your hand. Any type of communication among the participants during the session is forbidden and subject to the immediate expulsion from the experiment.

Introduction

You are one of the two firms that compete in a market. The other firm is also one of the participants in the session. Both firms are randomly matched and participate in a market under the same conditions of demand and costs.

[13] Using the optimal interior choice $g^0 = (1 - \frac{1}{t}) \cdot (1 + \frac{c}{r})$, we get $\partial g^0 / \partial r < 0$, which implies choosing lower g's for higher r's, as long as one's t is not too high for any risk to be preferred to the certain outcome c (given the obvious restriction $g^0 \leq 1$), in which case a subject might choose $g = 1$ in panels involving different risk premia. In fact, the necessary condition for any risk to be preferred to the certain payoff is that $r > c \cdot (t - 1)$.

Demand

The computer simulates the behavior of 36 consumers distributed equally among 3 locations of a linear city. Therefore, in the market there is a population of 36 potential consumers. Each consumer buys just one unit of the product to the cheapest firm, taking into account not only the price but also the transportation costs. For the consumer, to buy one unit implies an expenditure that is equal to the sum of the price (p) payed plus a cost generated from being located a certain distance (d) from the firm that sells it. In other words:

Total expenditure for the consumer = Price of the product (p) + distance between the consumer and the firm (d)

In case of a tie (i.e. the expenditure generated for the consumer is the same independently of the firm from which she buys the product), the computer will divide the demand assigning the same number of consumers to each firm.

Decision making

In this experiment, you will make decisions during 25 periods. Each period, your decisions will be about:

- Your location (SEQ: *First, your location. Once both of you have made this decision and before proceding to the next decision, you will observe each other's location*)
- The price of your product (SEQ: *Second, the price of your product*)

Let us see some relevant details related with each one of those decisions:

Your location

In a linear city there are three possible locations in which you may locate as a firm: W (West), C (Center) and E (East). Location C is in 0, location E is far from the center 1,500 units of distance to the right, and location W is far from the center 1,500 units of distance to the left. In each of these three locations there are 12 consumers. The following figure illustrates this situation:

Insert Figure 1 about here

At the beginning of each round, you will decide where to locate and at which price you want to sell the product unit. (SEQ: *At the beginning of each round, you will first decide where to locate. After knowing the location of your rival, then you will decide on the price of your product.*)

Price of the product

The price for you product may be a entire number between 1 and 1,000 ECUs (both included). Location and price decisions will be taken at the same time, in the sense that will be fixed in the same computer screen. (SEQ: *Location and price decisions will be taken sequentially in two consecutive screens.*)

Summary

Each round you have to decide on your location as well as on the price for you product. (SEQ: *first on your location and, second, on the price for your product.*)

Take into account that before the OK key is pressed, you may change your decision as many times as you want. Only when you are sure about your decisions, please press the OK key in order for the experiment to go on.

Once the two firms have decided, the system will calculate how many consumers buy from you and your profits for that round.

Remember that a consumer will decide to buy from the firm for which the total expenditure is the lowest (total expenditure=price + transportation cost of going to buy the unit from her location to the location of the firm). For the consumer, each unit of distance between your location and that of the seller firm implies a loss of 1 monetary unit.

For example, if the consumer located in E buys from the firm located in C, and the price is 500 ECUs, the total expenditure from buying is equal to 2,000 ECUs (p+d=500+1,500=2,000).

Additional information

At the end of each round you will receive information in your screen related to:

- Your location and price for that specific round

- Your rivals location and price for that specific round

- Your demand and profits for that specific round

Therefore, at the end of each round you will only see the decisions and outcome of that specific round.

Earnings

At the end of the session, the system will choose randomly 5 rounds. You will be paid in cash an amount in euro that is equal to the sum of your profits in ECUs in those 5 rounds multiplied divided by 5,000. To this amount we will add a show up fee of 3 Euro.

Before the session starts, you will perform a comprehension test to ensure that you understood well the context of the decision making.

Thank you for your collaboration.

6.3 The case of prices ranging between 0 and 1000

We now consider that a) in each location there are exactly 12 consumers, b) prices can take values from 0 to 1000, c) distance between districts is properly scaled up to have the same degree of price competition intention as in the case analyzed in the formal part of the paper and d) $v(x) = \begin{cases} \frac{1-e^{-a\frac{x}{36000}}}{1-e^{-a}} & \text{if } a \neq 0 \\ \frac{x}{36000} & \text{if } a = 0 \end{cases}$.

Therefore our unique symmetric equilibrium takes the form:

$$\hat{F}_A^W(p) = \begin{cases} 0 & \text{if } p \in [0, p') \\ \frac{e^{a/3}\left(-1-e^{\frac{ap}{3000}}-e^{\frac{ap}{1500}}+e^{\frac{ap}{1000}}+2\left(e^{a\left(-\frac{2}{3}+\frac{p}{1000}\right)}+e^{a\left(-\frac{1}{3}+\frac{p}{1000}\right)}-e^{\frac{a(-500+p)}{1500}}\right)\right)}{\left(2+3e^{a/3}\right)\left(-1+e^{\frac{ap}{3000}}\right)\left(1+e^{\frac{ap}{3000}}\right)^2} & \text{if } p \in [p', 1000] \\ \frac{1}{3+2e^{-a/3}} & \text{if } p > 1000 \end{cases}$$

and

$$\hat{F}_A^C(p) = \begin{cases} 0 & \text{if } p \in [0, p'') \\ \frac{e^{-a/3}\left(-2e^{a/3}-e^{2a/3}+2e^{a\left(\frac{1}{3}+\frac{p}{1000}\right)}+e^{a\left(\frac{2}{3}+\frac{p}{1000}\right)}+2e^{\frac{ap}{1000}}-2e^{\frac{a(2000+p)}{3000}}\right)}{\left(2+3e^{a/3}\right)\left(-1+e^{\frac{ap}{1000}}\right)} & \text{if } p \in [p'', 1000] \\ 1-\frac{2}{3+2e^{-a/3}} & \text{if } p > 1000 \end{cases}$$

for $0 < p' < p'' < 1000$ which satisfy $\hat{F}_A^W(p') = 0$ and $\hat{F}_A^C(p'') = 0$.

In particular for the case of risk neutral firms, that is for $v(x) = \frac{x}{36000}$, our equilibrium significantly simplifies to

$$\hat{F}_A^W(p) = \begin{cases} 0 & \text{if } p \in [0, 500) \\ \frac{2}{5} - \frac{200}{p} & \text{if } p \in [500, 1] \\ \frac{1}{5} & \text{if } p > 1000 \end{cases}$$

and

$$\hat{F}_A^C(p) = \begin{cases} 0 & \text{if } p \in [0, \frac{4000}{13}) \\ \frac{13}{15} - \frac{800}{3p} & \text{if } p \in [\frac{4000}{13}, 1] \\ \frac{3}{5} & \text{if } p > 1000 \end{cases}.$$

In Table A.2 we present some brief information for our equilibrium for various values of the risk parameter a.

Insert Table A.2 about here

7 Figures

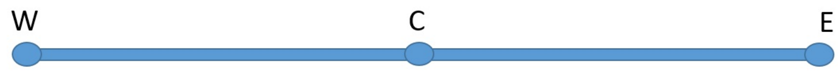

Figure 1: Linear city with three locations (West, Centre, East).

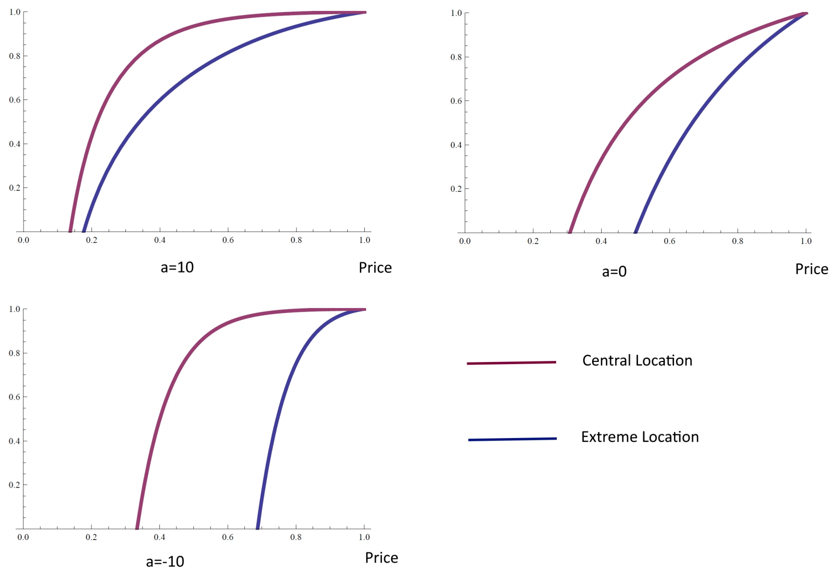

Figure 2: Conditional CDFs Price distributions for $a = 0, 10, -10$ in the unique symmetric equilibrium of the catalog competition game.

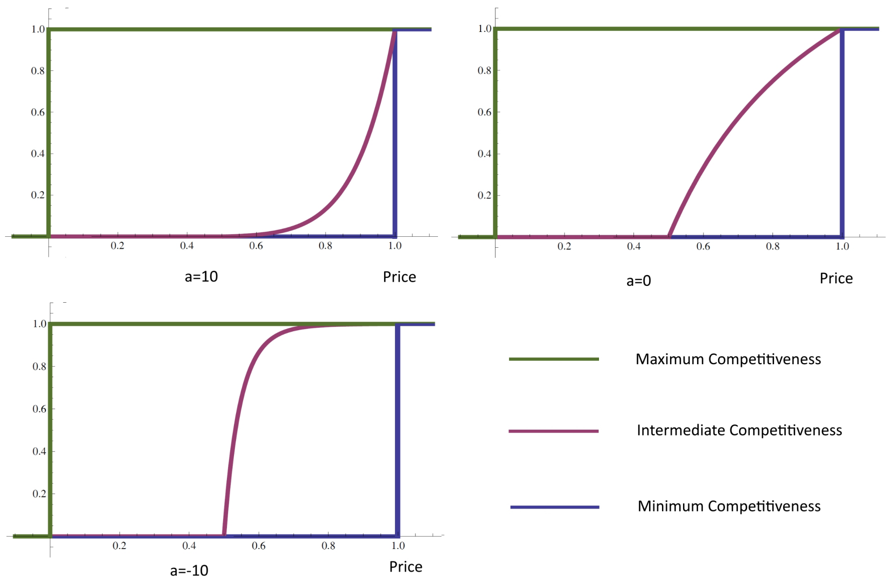

Figure 3: Conditional CDFs Price distributions for $a = 0, 10, -10$ in the unique symmetric equilibrium of the first-location-then-pricing game.

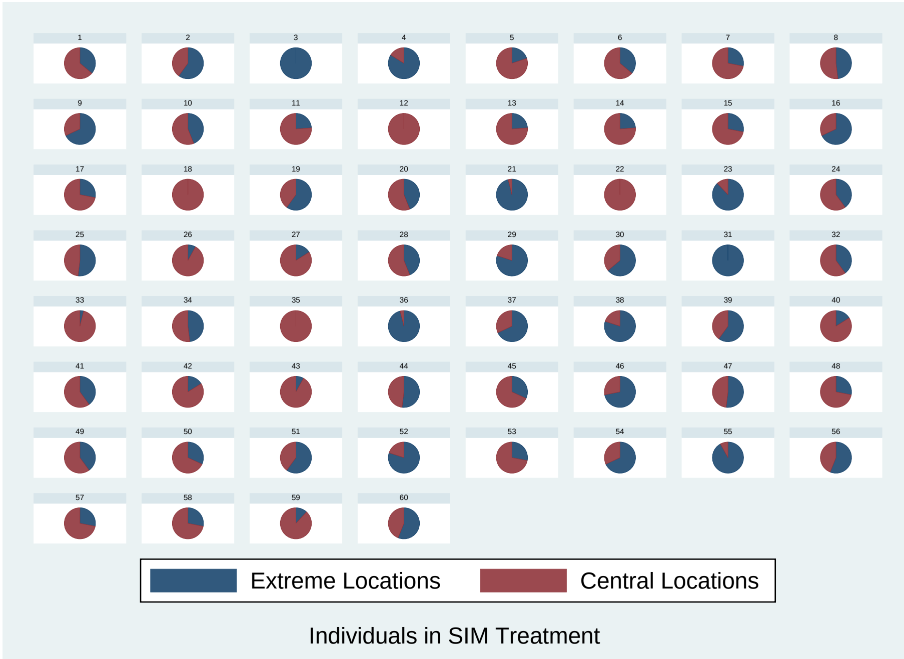

Figure 4: Individual location choices in SIM.

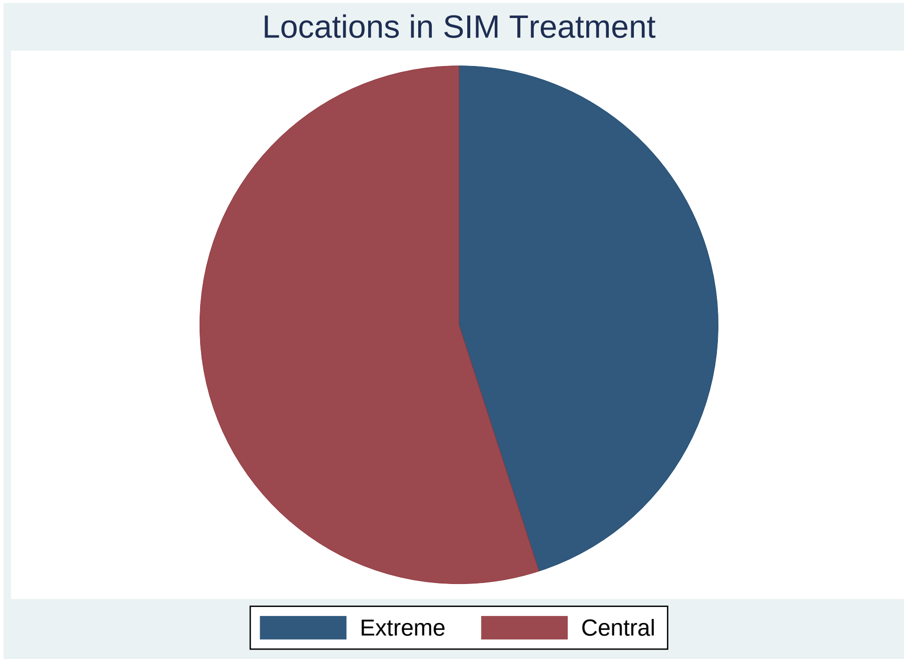

Figure 5: Aggregate location choices in SIM.

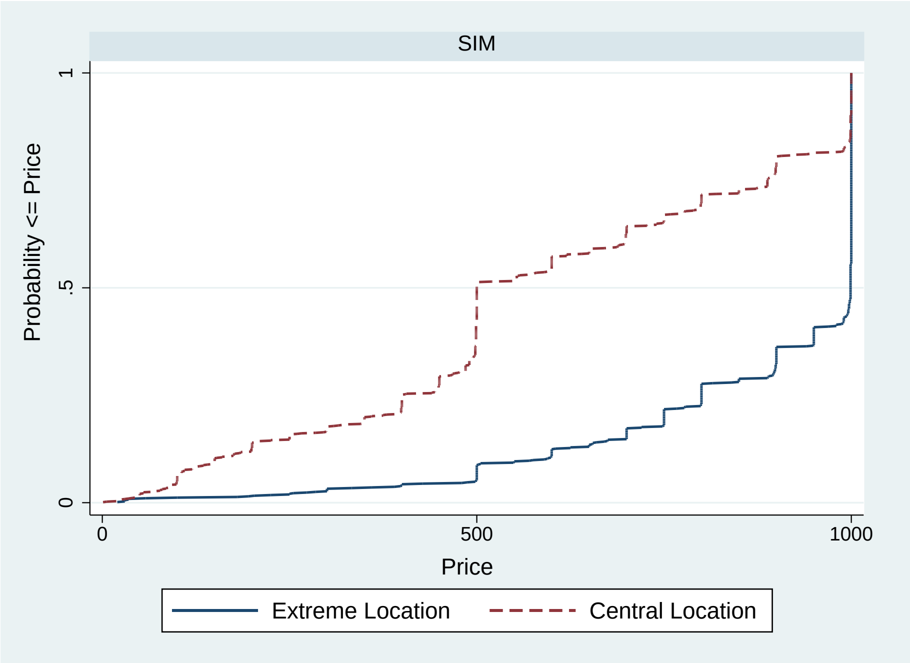

Figure 6: Price distribution in SIM by location strategy-profile.

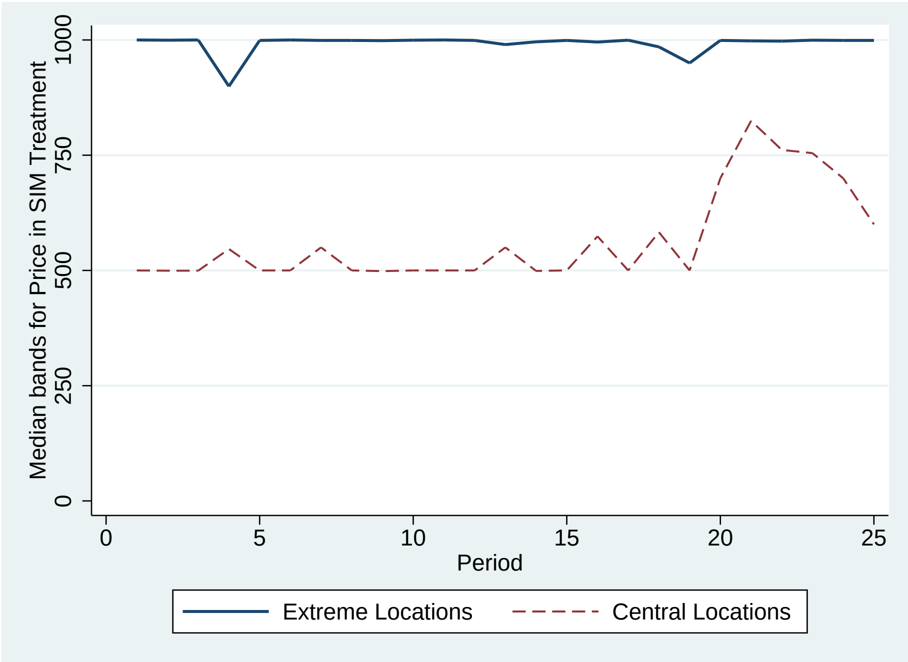

Figure 7: Evolution of prices by location strategy-profile in SIM.

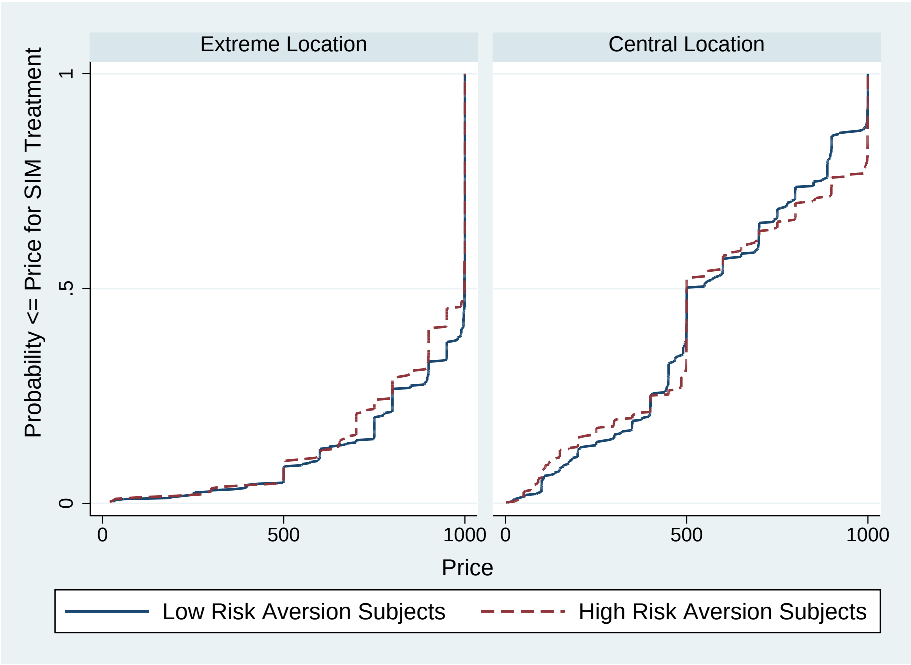

Figure 8: Price distribution by location and subject's risk attitude in SIM.

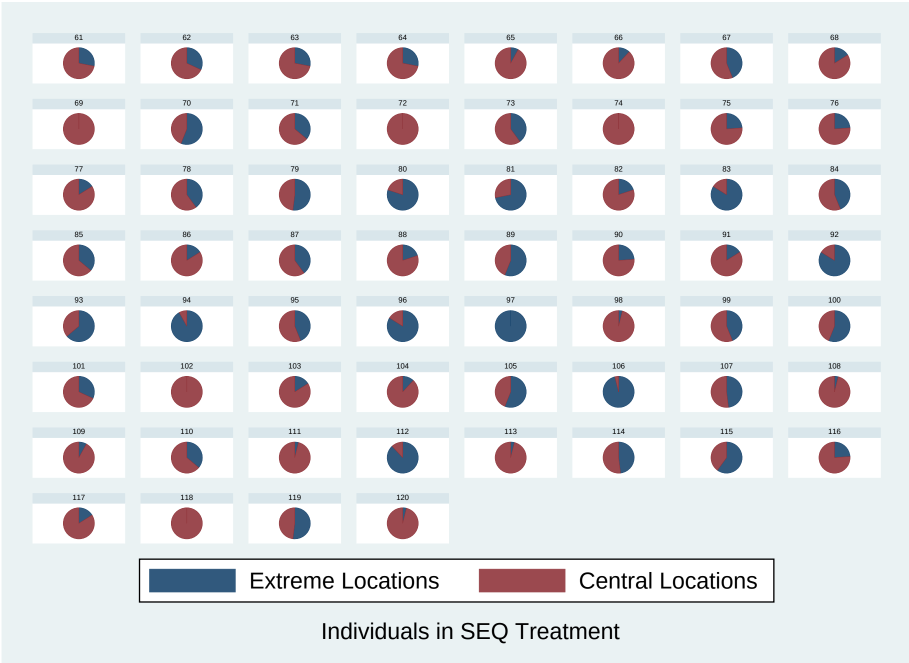

Figure 9: Individual location choices in SEQ.

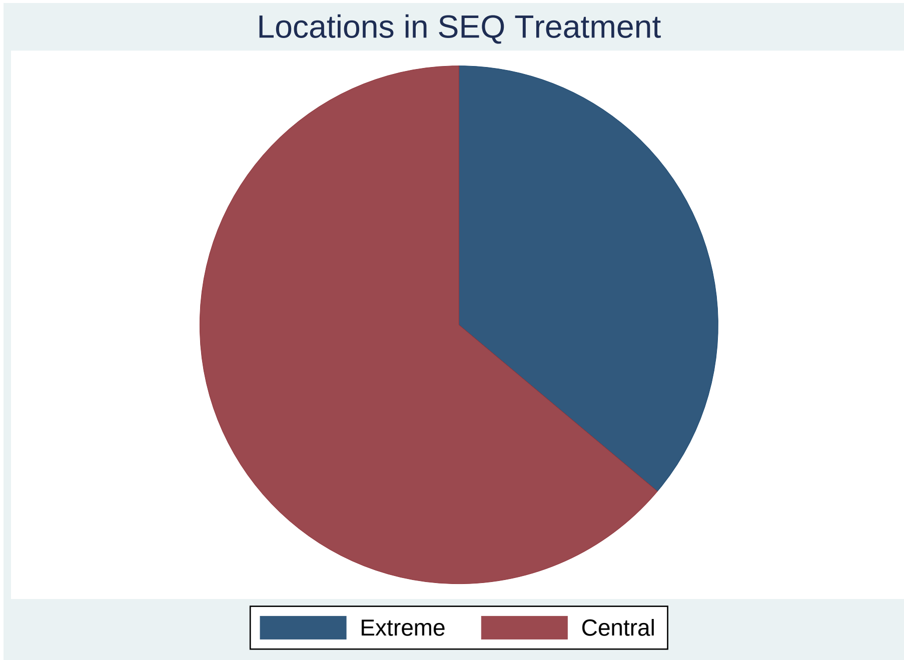

Figure 10: Aggregate location choices in SEQ.

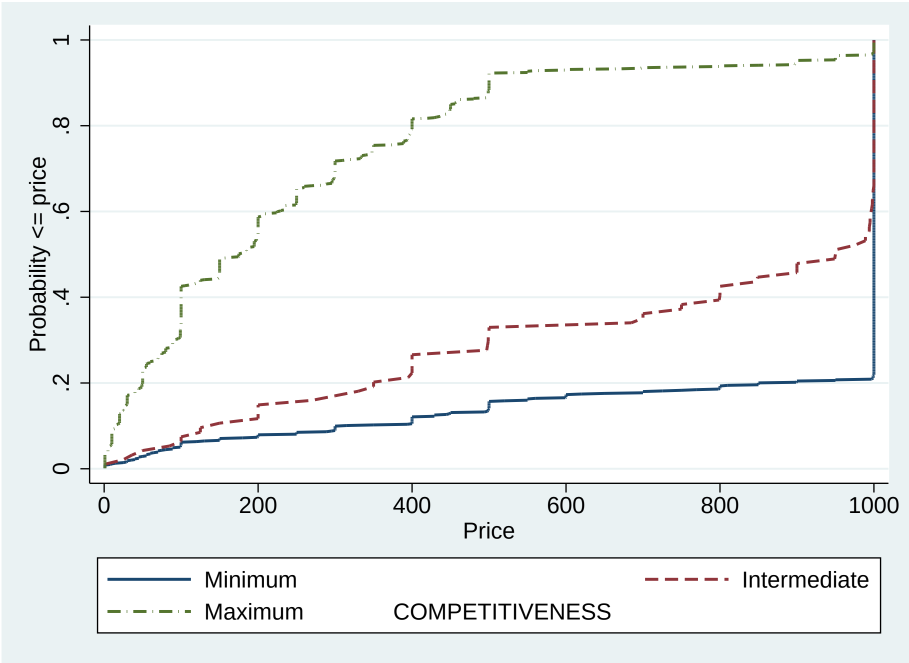

Figure 11: Price distritution by competitiveness in SEQ.

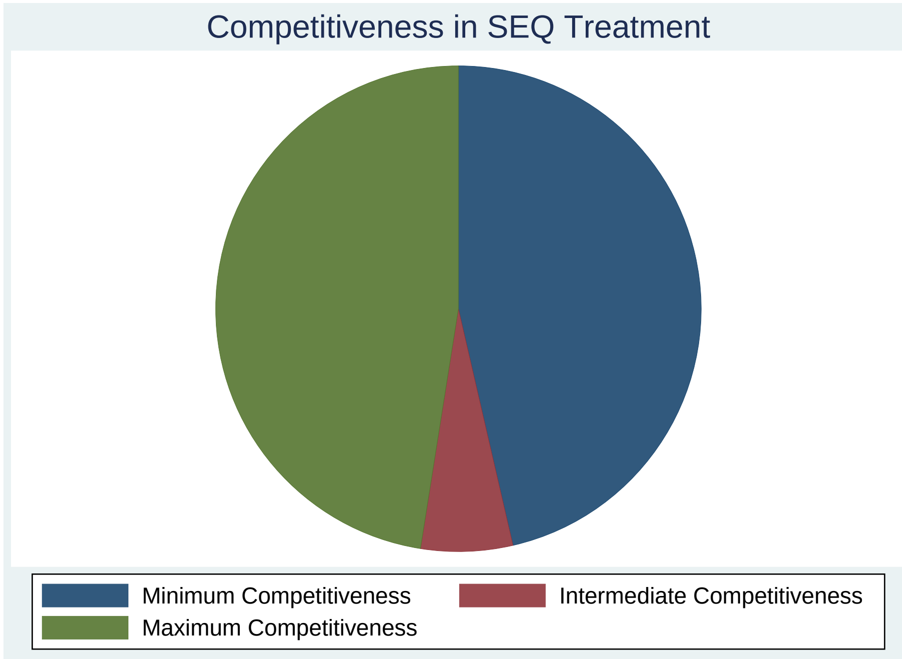

Figure 12: Aggregate location profiles in SEQ.

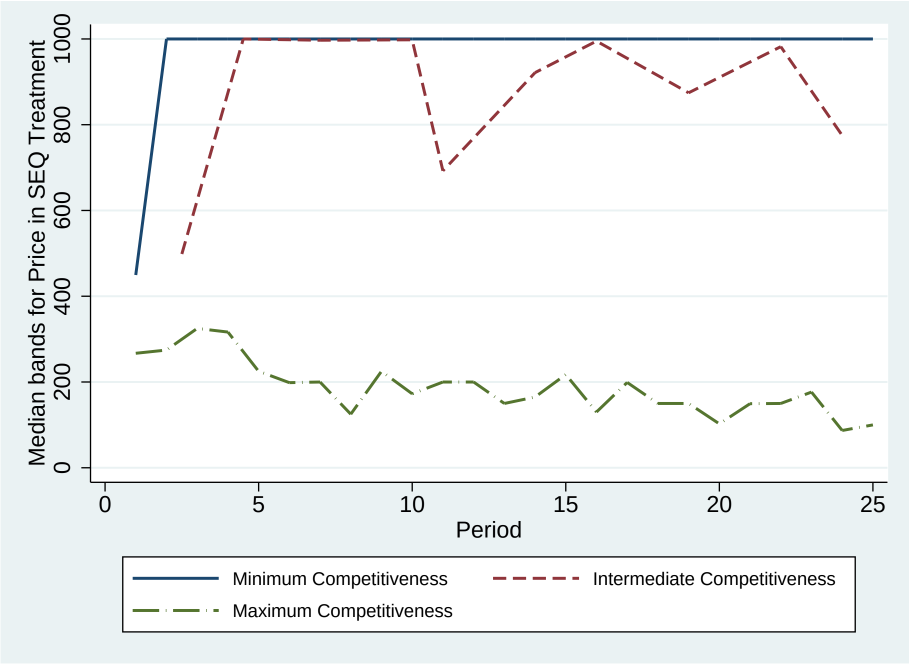

Figure 13: Evolution of prices by location strategy-profile in SEQ.

8 Tables

a	p'	p''	q
-10	0.677	0.338	0.01693
-1	0.54	0.320	0.17267
0	0.5	0.307	0.20000
1	0.46	0.290	0.22557
10	0.176	0.136	0.32559

Table 1: Equilibrium features for various levels of risk aversion.

	SIM				SEQ				Kruskall Wallis test	
Variable	**Mean**	**Std. Dev.**	**Min**	**Max**	**Mean**	**Std. Dev.**	**Min**	**Max**	**Chi2**	**p-value**
age	23.92	4.78	18	38	22.88	3.88	18	37	1.19	0.2796
female	0.50	0.50	0	1	0.48	0.50	0	1	0.03	0.8557
riskavers	5.53	2.34	0	9	5.38	2.23	0	9	0.38	0.5369
economics	0.65	0.48	0	1	0.65	0.48	0	1	0.00	1.0000
price	751.57	249.79	55	1,000	528.45	442.25	8	1,000	20.88	**0.0001**
location	1.60	0.49	1	2	1.63	0.49	1	2	3.37	**0.0666**
competitiveness	1.00	0.97	0	2	1.03	0.99	0	2	2.22	0.1367

Table 2: Descriptive statistics for the measured variables.

	Coef.	Std. Err.	z	P>z	95% Conf. Interval
Constant SIM	351.23	38.36	9.15	0.000	[276.04, 426.43]
Diff. Constant SEQ	496.90	36.90	13.46	0.000	[424.57, 569.23]
Lag. Competitiveness SIM	-40.65	6.06	-6.70	0.000	[-52.54, -28.75]
Competitiveness SEQ	-316.70	7.40	-42.80	0.000	[-331.21, -302.20]
Riskavers SIM	-8.84	3.21	-2.75	0.006	[-15.14,-2.54]
Diff. Riskavers SEQ	-8.80	3.65	-2.41	0.016	[-15.97, -1.63]
Lag. Price SIM	0.56	0.046	12.21	0.000	[0.471, 0.656]
Diff. Lag. Price SEQ	-0.39	0.03	-13.21	0.000	[-0.456, -0.338]
Period SIM	2.62	0.937	2.80	0.005	[0.789, 4.465]
Diff. Period SEQ	-2.01	1.31	-1.54	0.124	[-4.583, 0.552]

Table 3: Prais-Winsten regression, correlated panels corrected standard errors (PCSEs) for Price. Group variable: subjectid; Time variable: period; Number of obs = 2,880; Number of groups = 120; Panels: correlated (balanced); Autocorrelation: no; Obs. per group: 24; Estimated covariances = 7,260; Estimated autocorrelations = 0; Estimated coefficients = 10; R^2 = 0.5647; Wald $\chi^2(9)$ =2594.37; Prob > χ^2 = 0.0000;

Panel 1

g	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
X in Euros	1.00	1.12	1.27	1.47	1.73	2.10	2.65	3.56	5.40	10.90

Panel 2

g	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
X in Euros	1.00	1.20	1.50	1.90	2.30	3.00	4.00	5.70	9.00	19.00

Panel 3

g	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
X in Euros	1.00	1.66	2.50	3.57	5.00	7.00	10.00	15.00	25.00	55.00

Panel 4

g	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
X in Euros	1.00	2.20	3.80	5.70	8.30	12.00	17.50	26.70	45.00	100

Table A.1: *Lottery Panels* risk elicitation task. Subjects choose their preferred lottery from each panel.

a	p'	p''	q
-10	677	338	0.01693
-1	540	320	0.17267
0	500	307	0.20000
1	460	290	0.22557
10	176	136	0.32559

Table A.2: Equilibrium features for various levels of risk aversion when price ranges between 0 and 1000.