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UNIVERSITAT JAUME I

Which estimator to measure local governments' cost efficiency? An application to Spanish municipalities

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2017 / 06

Castellón (Spain)

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Abstract

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April 8, 2017

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1. Introduction

Managing the available resources efficiently at all levels of government (central, regional, and municipal) is essential, particularly in the scenario of the current international economic crisis, which still affects several European countries. Given that increasing taxes and deficit is politically costly (Doumpos and Cohen, 2014), a reasonable way to operate in this context is to improve economic efficiency (De Witte and Geys, 2011), which in cost terms means that an entity should produce a particular level of output in the cheapest way. In this setting, since local regulators must provide the best possible local services at the lowest possible cost, developing a system for evaluating local government performance that allows benchmarks to be set over time could have relevant practical implications (Da Cruz and Marques, 2014). However, measuring the performance of local governments is usually highly complex.

Local government efficiency has attracted much scholarly interest in the field of public administration and there is now a large body of literature covering several countries, such as Balaguer-Coll et al. (2007) in Spain, Geys et al. (2013) in Germany or Štastná and Gregor (2015) in the Czech Republic, among others.[1] However, despite the high number of empirical contributions, a major challenge to analysis of local government performance is the lack of clear, standard methodology to perform efficiency analysis. This is not a trivial question as much previous literature has proposed different frontier techniques, both parametric and non-parametric, to analyse technical, cost or other forms of efficiency in local governments.

Although this problem is well-known in the efficiency measurement literature, few studies have attempted to use two or more alternative approaches comparatively. For instance, De Borger and Kerstens (1996a) analysed local governments in Belgium using five different reference technologies, two non-parametric (Data Envelopment Analysis or DEA, and Free Disposal Hull or FDH) and three parametric frontiers (one deterministic and two stochastic). They found large differences in the efficiency scores for identical samples and, as a consequence, suggested using different methods to control for the robustness of results whenever the problem of choosing the "best" reference technology is unsolved. Other studies compared the efficiency estimates of DEA and Stochastic Frontier Approach (SFA),[2] or DEA and FDH or

[1] For a comprehensive literature review on efficiency measurement in local governments see Narbón-Perpiñá and De Witte (2017a,b).

[2] Athanassopoulos and Triantis (1998); Worthington (2000); Geys and Moesen (2009b); Boetti et al. (2012); Nikolov and Hrovatin (2013); Pevcin (2014)

other non-parametric variants,[3] and drew similar conclusions.

Since there is no obvious way to choose an efficiency estimator, the method selected may affect the efficiency analysis (Geys and Moesen, 2009b) and could lead to biased results. Therefore, if local government decision makers set a benchmark based on an incorrect efficiency score, a non-negligible economic impact may result. Accordingly, as Badunenko et al. (2012) point out, if the selected method overestimates the efficiency scores, some local governments may not be penalised and, as a result, their inefficiencies will persist. In contrast, if the efficiency scores are underestimated some local governments would be regarded as "low performers" and could be unnecessarily penalised. Hence, although we note that each particular methodology leads to different cost efficiency results for each local government, one should ideally report efficiency scores that will be more reliable, or closer to the truth (Badunenko et al., 2012).[4]

The present investigation addresses these issues by comparing four non-parametric methodologies and uncovering which measures might be more appropriate to assess local government cost efficiency in Spain. The study contributes to the literature in three specific aspects. First, we seek to compare four non-parametric methodologies that cover traditional and recently developed non-parametric frameworks, namely DEA, FDH, the order-m partial frontier (Cazals et al., 2002) and the bias-corrected DEA estimator proposed by Kneip et al. (2008); the first two are the most popular in the non-parametric field while the latter two are more recent proposals. These techniques have been widely studied in the previous literature, but little is known about their performance in comparison with each other. Indeed, this is the first study that compares these efficiency estimators between them.

Second, we attempt to determine which of these methods should be applied to measure cost efficiency in a given situation. In contrast to previous literature, which has regularly compared techniques and made alternative proposals, we follow the method set out by Badunenko et al. (2012), with the aim to compare the different methods used and identify those that perform better in different settings. We carry out the experiment via Monte Carlo simulations and discuss the relative performance of the efficiency estimators under various scenarios.

Our final contribution is to identify which methodologies perform better with our par-

[3] Balaguer-Coll et al. (2007); Fogarty and Mugera (2013); El Mehdi and Hafner (2014)

[4] We will elaborate further on this *a priori* ambitious expression.

ticular dataset. From the simulation results, we determine in which scenario our data lies in, and follow the suggestions related to the performance of the estimators for this scenario. Therefore, we use a consistent method to choose an efficiency estimator, which provides a significant contribution to previous literature in local government efficiency. We use a sample of 1,574 Spanish local governments of municipalities between 1,000 and 50,000 inhabitants for the period 2008–2013. While other studies based on Spanish data (as well as data from other countries) focus on a specific region or year, our study examines a much larger sample of Spanish municipalities comprising various regions for several years.

The sample is also relevant in terms of the period analysed. The economic and financial crisis that started in 2007 has had a huge impact on most Spanish local government revenues and finances in general. In addition, the budget constraints became stricter with the law on budgetary stability,[5] which introduced greater control over public debt and public spending. Under these circumstances, issues related to Spanish local government efficiency have gained relevance and momentum. Evaluation techniques give the opportunity to identify policy programs that are working well, to analyse aspects of a program that can be improved, and to identify other public programs that do not meet the stated objectives. In fact, gaining more insights into the amount of local government inefficiency might help to further support effective policy measures to correct and or control it. Therefore, it is obvious that obtaining here a reliable efficiency score would have relevant economic and political implications.

Our results suggest that there is no one approach suitable for all efficiency analysis. When using these results for policy decisions, local regulators must be aware of which part of the distribution is of particular interest and if the interest lies in the efficiency scores or the rankings estimates. We find that for our sample of Spanish local governments, all methods showed some room for improvement in terms of possible cost efficiency gains, however they present large differences in the inefficiency levels. Both DEA and FDH methodologies showed the most reliable efficiency results, according to the findings of our simulations. Therefore, our results indicate that the average cost efficiency would have been between 0.54 and 0.77 during the period 2008–2013, suggesting that Spanish local governments could have achieved the same level of local outputs with about 23% to 36% fewer resources. From a technical point of view, the analytical tools introduced in this study would represent an interesting contribution

[5] *Ley General Presupuestaria* (2007,2012), or General Law on the Budget.

that examine the possibility of using a consistent method to choose an efficiency estimator, and the obtained results give evidence on how efficiency could certainly be assessed to provide some additional guidance for policy makers.

The paper is organised as follows: section 2 gives an overview of the methodologies applied to determine the cost efficiency. Section 3 describes the data used. Section 4 shows the methodological comparison experiment and the results for the different scenarios. Section 5 suggests which methodology performs better with our dataset and presents and comments on the most relevant efficiency results. Finally, section 6 summarises the main conclusions.

2. Methodologies

In this section, we present our four different non-parametric techniques to measure cost efficiency[6], namely, DEA, FDH, order-m and Kneip et al.'s (2008) bias-corrected DEA estimator, which we will refer to as KSW.

2.1. Data Envelopment Analysis (DEA) and Free Disposal Hull (FDH)

DEA (Charnes et al., 1978; Banker et al., 1984) is a non-parametric methodology based on linear programming techniques to define an empirical frontier which creates an "envelope" determined by the efficient units. We consider an input-oriented DEA model because public sector outputs are established externally (the minimum services that local governments must provide) and it is therefore more appropriate to evaluate efficiency in terms of the minimisation of inputs (Balaguer-Coll and Prior, 2009).

We introduce the mathematical formulation for the cost efficiency measurement (Färe et al., 1994). The minimal cost efficiency can be calculated by solving the following program for each local government and each sample year:

[6]Different types of efficiency can be distinguished, depending on the data available for inputs and outputs: *technical efficiency* (TE) requires data on quantities of inputs and outputs, while *allocative efficiency* (AE) requires additional information on input prices. When these two measures are combined, we obtain the *economic efficiency*, also called *cost efficiency* $(CE = TE \cdot AE)$. In this paper, we measure local government cost efficiency since we have information relative to specific costs, although it is not possible to decompose it into physical inputs and input prices.

$$
\begin{aligned}
& min_{\theta,\lambda}\theta \\
& \quad s.t. \quad && y_{ri} \leq \textstyle\sum_{i=1}^{n} \lambda_i y_{ri}, && r = 1, \ldots, p \\
& && \theta x_{ji} \geq \textstyle\sum_{i=1}^{n} \lambda_i x_{ji}, && j = 1, \ldots, q \\
& && \lambda_i \geq 0, && i = 1, \ldots, n \\
& && \textstyle\sum_{i=1}^{n} \lambda_i = 1
\end{aligned}
\tag{1}
$$

where for n observations there are q inputs producing p outputs. The $n \times p$ output matrix, r, and the $n \times q$ input matrix, j, represent the data for all n local governments. Specifically, for each unit under evaluation i we consider an input vector x_{ji} to produce outputs y_{ri}. The last constraint ($\sum_{i=1}^{n} \lambda_i = 1$) implies variable returns to scale (VRS), which ensures that each DMU is compared only with others of similar sizes.

A further extention of the DEA model is the Free Disposal Hull (FDH) estimator proposed by Deprins et al. (1984). The main difference between DEA and FDH is that the latter drops the convexity assumption. FDH cost efficiency is defined as follows:

$$
\begin{aligned}
& min_{\theta,\lambda}\theta \\
& \quad s.t. \quad && y_{ri} \leq \textstyle\sum_{i=1}^{n} \lambda_i y_{ri}, && r = 1, \ldots, p \\
& && \theta x_{ji} \geq \textstyle\sum_{i=1}^{n} \lambda_i x_{ji}, && j = 1, \ldots, q \\
& && \lambda_i \in \{0, 1\}, && i = 1, \ldots, n \\
& && \textstyle\sum_{i=1}^{n} \lambda_i = 1
\end{aligned}
\tag{2}
$$

Finally, the solution of mathematical linear programming problems (1) and (2) yields optimal values for the cost efficiency coefficient θ. Local governments with efficiency scores of $\theta < 1$ are inefficient, while efficient units receive efficiency scores of $\theta = 1$.

2.2. Robust variants of DEA and FDH

The traditional non-parametric techniques DEA and FDH have been widely applied in efficiency analysis; however, it is well-known that they present several drawbacks, such as the influence of extreme values and outliers, the "curse of dimensionality"[7] or the difficulty of drawing classical statistical inference. Hence, we also consider two alternatives to DEA and

[7] An increase in the number of inputs or outputs, or a decrease in the number of units for comparison, implies higher efficiencies Daraio and Simar (2007).

FDH estimators that are able to overcome most of these drawbacks. The first is order-m (Cazals et al., 2002), a partial frontier approach that mitigates the influence of outliers and the curse of dimensionality, and the second is Kneip et al.'s (2008) bias-corrected DEA estimator (KSW), which allows for consistent statistical inference by applying bootstrap techniques.

2.2.1. Order-m

Order-m frontier (Cazals et al., 2002) is a robust alternative to DEA and FDH estimators that involves the concept of partial frontier. The order-m estimator, for finite m units, does not envelope all data points and is consequently less extreme. In the input orientation case, this method uses as a benchmark the expected minimum level of input achieved among a fixed number of m local governments producing at least output level y (Daraio and Simar, 2007). The value m represents the number of potential units against which we benchmark the analysed unit. Hence, the order-m input efficiency score is given by:

$$\hat{\theta}_m(x,y) = E[(\hat{\theta}_m(x,y)|Y \geqslant y)] \tag{3}$$

If m goes to infinity, the order-m estimator converges to FDH. The most reasonable value of m is determined as the value for which the super-efficient observations becomes constant (Daraio and Simar, 2005). Note that order-m scores are not bounded by 1 as DEA or FDH. A value greater than 1 indicates super-efficiency, showing that the unit operating at the level (x,y) is more efficient than the average of m peers randomly drawn from the population of units producing more output than y (Daraio and Simar, 2007).

2.2.2. Kneip et al.'s (2008) bias-corrected DEA estimator (KSW)

The KSW (Kneip et al., 2008) is a bias-corrected DEA estimator which derives the asymptotic distribution of DEA via bootstrapping techniques. Simar and Wilson (2008) noted that DEA and FDH estimators are biased by construction, implying that the true frontier would lie under the DEA estimated frontier. Badunenko et al. (2012) explained that, the bootstrap procedure to correct this bias, based on sub-sampling, "uses the idea that the known distribution of the difference between estimated and bootstrapped efficiency scores mimics the unknown distribution of the difference between the true and the estimated efficiency scores".

This procedure provides consistent statistical inference of efficiency estimates (i.e., bias and confidence intervals for the estimated efficiency scores).

In order to implement the bootstrap procedure (based on sub-sampling), first let $s = n^d$ for some $d \in (0, 1)$, where n is the sample size and s is the sub-sample size. Then, the bootstrap is outlined as follows:

1. First, a bootstrap sub-sample $S_s^* = (X_i^*, Y_i^*)_{i=1}^s$ is generated by randomly drawing (independently, uniformly and with replacement) s observations from the original sample, S_n.

2. Apply the DEA estimator, where the technology set is constructed with the sub-sample drawn in step (1), to construct the bootstrap estimates $\hat{\theta}^*(x, y)$.

3. Steps (1) and (2) are repeated B times, which allows approximation of the conditional distribution of $s^{2/(p+q+1)}(\frac{\hat{\theta}^*(x,y)}{\theta^*(x,y)} - 1)$ and the unknown distribution of $n^{2/(p+q+1)}(\frac{\hat{\theta}^*(x,y)}{\theta^*(x,y)} - 1)$. The values p and q are the output and input quantities, respectively. The bias-corrected DEA efficiency score is given by:

$$\theta_{bc}(x, y) = \theta^*(x, y) - Bias^* \tag{4}$$

   where the bias is adjusted by employing the s sub-sample size.

$$Bias^* = \left(\frac{s}{n}\right)^{2/(p+q+1)} \left[\frac{1}{B}\sum_{b=1}^{B} \hat{\theta}_b^*(x, y) - \theta^*(x, y)\right] \tag{5}$$

4. Finally, for a given $\alpha \in (0, 1)$, the bootstrap values are used to find the quantiles $\delta_{\alpha/2,s}$, $\delta_{1-\alpha/2,s}$ in order to compute a symmetric $1 - \alpha$ confidence interval for $\theta(x, y)$

$$\left[\frac{\hat{\theta}(x, y)}{1 + n^{-2/(p+q+1)}\delta_{1-\alpha/2,s}}, \frac{\hat{\theta}(x, y)}{1 + n^{-2/(p+q+1)}\delta_{\alpha/2,s}}\right] \tag{6}$$

3. Sample, data and variables

We consider a sample of Spanish local governments of municipalities between 1,000 and 50,000 inhabitants for the 2008–2013 period. The information on inputs and outputs was obtained from the Spanish Ministry of the Treasury and Public Administrations (*Ministerio de Hacienda y Administraciones Públicas*). Specific data on outputs were obtained from a survey on local infrastructures and facilities (*Encuesta de Infraestructuras y Equipamientos Locales*). Information on inputs was obtained from local governments' budget expenditures. The final sample contains 1,574 Spanish municipalities for every year, after removing all the observations for which information on inputs or outputs was not available for the sample period (2008–2013).

Inputs are representative of the cost of the municipal services provided. Using budget expenditures as inputs is consistent with previous literature (e.g., Balaguer-Coll et al., 2007, 2010; Zafra-Gómez and Muñiz-Pérez, 2010; Fogarty and Mugera, 2013; Da Cruz and Marques, 2014). We construct an input measure, representing total local government costs (X_1), that includes municipal expenditures on personnel expenses, expenditures on goods and services, current transfers, capital investments and capital transfers.

Outputs are related to the minimum specific services and facilities provided by each municipality. Our selection is based on article 26 of the Spanish law which regulates the local system (*Ley reguladora de Bases de Régimen Local*). It establishes the minimum services and facilities that each municipality is legally obliged to provide, depending on their size. Specifically, all governments must provide public street lighting, cemeteries, waste collection and street cleaning services, drinking water to households, sewage system, access to population centres, paving of public roads, and regulation of food and drink. The selection of outputs is consistent with the literature (e.g., Balaguer-Coll et al., 2007; Balaguer-Coll and Prior, 2009; Zafra-Gómez and Muñiz-Pérez, 2010; Bosch-Roca et al., 2012). Note that in contrast to previous studies in other European countries, we do not include outputs such as the provision of of primary and secondary education, care for the elderly or health services, since they do not fall within the responsibilities of Spanish municipalities.

As a result, we chose six output variables to measure the services and facilities municipalities provide. Due to the difficulties in measuring public sector outputs, in some cases it is necessary to use proxy variables for the services delivered by municipalities given the unavailability of more direct outputs (De Borger and Kerstens, 1996a,b), an assumption which has been widely applied in the literature. Table 1 reports the minimum services that all local government were obliged to provide for the 2008–2013 period, as well as the output indicators used to evaluate the services. Table 2 reports descriptive statistics for inputs and outputs for the same period. We include the median instead of the mean in an attempt to avoid distortion by outliers.

4. Methodological comparison

In contrast to the previous literature, in this section we compare DEA, FDH, order-m and KSW approaches following the method proposed by Badunenko et al. (2012).[8] Our aim is to uncover which measures perform best with our particular dataset, that is, which ones are the most appropriate to measure local government efficiency in Spain in order to provide useful information for local governments' performance decisions.

To this end, we carry out the experiment via Monte Carlo simulations. We first define the data generating process, the parameters and the distributional assumptions on data. Second, we consider the different methodologies and take several standard measures to compare their behaviour. Next, after running the simulations, we discuss the relative performance of the efficiency estimators under the various scenarios. Finally, we decide which methods are the most appropriate to measure local government efficiency in Spain.

4.1. Simulations

Several previous studies analysing local government cost efficiency with parametric techniques used the SFA estimator developed byAigner et al. (1977) and Meeusen and Van den Broeck (1977) as a model to estimate cost frontiers.[9] These studies considered the input-oriented efficiency where the dependent variable is the level of spending or cost, and the independent variables are output levels. As a parametric approach, SFA establishes the best practice frontier on the basis of a specific functional form, most commonly Cobb-Douglas

[8]The study of Badunenko et al. (2012) compared two estimators of technical efficiency in a cross-sectional setting. Specifically, they compared SFA, represented by the non-parametric kernel SFA estimator of Fan et al. (1996), with DEA, represented by the non-parametric bias-corrected DEA estimator of Kneip et al. (2008).

[9]See, for instance, the studies of Worthington (2000), De Borger and Kerstens (1996a), Geys (2006), Ibrahim and Salleh (2006), Geys and Moesen (2009a,b), Kalb (2010), Geys et al. (2010), Kalb et al. (2012) or Šťastná and Gregor (2015), Lampe et al. (2015), among others.

or Translog. Moreover, it allows researchers to distinguish between measurement error and inefficiency term.

Following this scheme, we conduct simulations for a production process with one input or cost (c) and two outputs (y_1 and y_2).[10] We consider a Cobb-Douglas cost function (CD). For the baseline case, we assume constant returns to scale (CRS) ($\gamma = 1$).[11] We establish $\alpha = 1/3$ and $\beta = \gamma - \alpha$.

We simulate observations for outputs y_1 and y_2, which are distributed uniformly on the $[1, 2]$ interval. Moreover, we assume that the true error term (v) is normally distributed $N(0, \sigma_v^2)$ and the true cost efficiency is $TCE = exp(-u)$, where u is half-normally distributed $N^+(0, \sigma_u^2)$ and independent from v. We introduce the true error and inefficiency terms in the frontier formulation, which takes the following expression:

$$c = y_1^\alpha \cdot y_2^\beta \cdot exp(v + u), \tag{7}$$

where c is total costs and y_1 and y_2 are output indicators. For reasons explained in section 2, there is no observable variation in input prices, so input prices are ignored (see, for instance, the studies of Kalb, 2012, and Pacheco et al., 2014).

We simulate six different combinations for the error and inefficiency terms, in order to model various real scenarios. Table 3 contains the matrix of the different scenarios. It shows the combinations when σ_v takes values 0.01 and 0.05 and σ_u takes values 0.01, 0.05 and 0.1. The rows in the table represent the variation of the error term (σ_v), while the columns represent the variation of the inefficiency term (σ_u). The first row is the case where the variation of the error term is relatively small, while the second row shows a large variation. The first column is the case where the inefficiency term is relatively small, while the second and third columns represent the cases where variation in inefficiency is relatively larger. The Λ parameter, which sets each scenario, is the ratio between of σ_u and σ_v.

Within this context, scenario 1 is the case when the error and the inefficiency terms are relatively small ($\sigma_u = 0.01$, $\sigma_v = 0.01$, $\Lambda = 1.0$), which means that the data has been measured with little noise and the units are relatively efficient, while scenario 6 is the case when the

[10] For simplicity, we use a multi-output model with two outputs instead of six.

[11] In subsection 4.4, we consider robustness checks with increasing and decreasing returns to scale to make sure that our simulations accurately represent the performance of our methods.

error and the inefficiency terms are relatively large ($\sigma_u = 0.1$, $\sigma_v = 0.05$, $\Lambda = 2.0$), which means that the data is relatively noisy and the units are relatively inefficient.

For all simulations we consider 2,000 Monte Carlo trials, and we analyse two different sample sizes, n= 100 and 200.[12] We note that non-parametric estimators do not take into account the presence of noise, however, we want to check how it affects the performance of our estimators since all data tend to have noise.[13]

4.2. Measures to compare the estimators' performance

In order to compare the relative performance of our four non-parametric methodologies, we consider the following median measures over the 2,000 simulations. We use median values instead of the average, since it is more robust to skewed distributions.

- $Bias(TCE) = \frac{1}{n}\sum_{i=1}^{n}(\widehat{TCE_i} - TCE_i)$
- $RMSE(TCE) = [\frac{1}{n}\sum_{i=1}^{n}(\widehat{TCE_i} - TCE_i)^2]^{1/2}$
- $UpwardBias(TCE) = \frac{1}{n}\sum_{i=1}^{n} 1 \cdot (\widehat{TCE_i} > TCE_i)$
- Kendall's τ (TCE)= $\frac{n_c - n_d}{0.5n(n-1)}$

where $\widehat{TCE_i}$ is the estimated cost efficiency of municipality i in a given Monte Carlo replication (by a given method) and TCE_i is the true efficiency score. The bias reports the difference between the estimated and true efficiency scores. When it is negative (positive), the estimators are underestimating (overestimating) the true efficiency. The $RMSE$ (root mean squared error) measures the standard deviation or error from the true efficiency. The upward bias is the proportion of $\widehat{TCE}$ larger than the true efficiencies. It measures the percentage of overestimated or underestimated cost efficiencies. Finally, the Kendall's τ test represents the correlation between the predicted and true cost efficiencies, where n_c and n_d are the number of concordant and discordant pairs in the data set, respectively. This test identifies the differences in the ranking distributions of the true and the estimated ranks.

[12]To ease the computational process, we use samples of n= 100 and 200 to conduct simulations. In subsection 4.4, we consider a robustness check with a bigger sample size ($n = 500$) to ensure that our simulations accurately represent the performance of our data.

[13]In subsection 4.4, we consider a robustness check with no noise to ensure that our simulations accurately represent the performance of our data.

We also compare the densities of cost efficiency across all Monte Carlo simulations in order to report a more comprehensive description of the results, not only restrict them to a single summary statistic—the median. So, for example, if we were interested in estimating the poorer performers, we would focus on which estimator perform best at the 5*th* percentile of the efficiency distribution. For each draw, we sort the data by the relative value of true efficiency. Since we are interested in comparing the true distribution for different percentiles of our sample, we show violin plots for 5%, 50% and 95% percentiles.

4.3. Relative performance of the estimators

Table 4 provides baseline results for the performance measures of the cost efficiency with the CD cost function. First we observe that the median bias of the cost efficiency scores is negative in DEA and KSW in all cases. This implies that the DEA and KSW estimators tend to underestimate the true cost efficiency in all scenarios. FDH and order-m present positive median bias except for scenario 2 in FDH, implying a tendency to overestimate the true efficiency. Bias for all methodologies tends to increase with the sample size when the bias is negative, and decrease when the bias is positive, except for order-m in scenarios 1, 3 and 5. The RMSE is smaller when σ_v is small, except for FDH in scenario 5 and order-m in scenarios 3 and 5. Moreover, the RMSE of the cost efficiency estimates increases with the sample size for all cases except for FDH in scenarios 1, 3, 5 and 6 and order-m in scenarios 5 and 6.

We also consider the upward bias. This shows the percentage of observations for which cost efficiency is larger than the true value (returning a value of 1). The desired value is 0.5. The values less (greater) than 0.5 indicate underestimation (overestimation) of cost efficiencies. In this setting, DEA and KSW systematically underestimate the true efficiency. Moreover, as the sample size increases, so does the percentage of underestimated results. In contrast, FDH and order-m tend to overestimate the true efficiency, but as the sample size increases overestimated results decrease. Finally, we analyse Kendall's τ for the efficiency ranks between true and estimated efficiency scores. In each scenario and sample size, DEA and KSW have a larger Kendall's τ; they therefore perform best at identifying the ranks of the efficiency scores.

We also analyse other percentiles of the efficiency distribution, since it is difficult to conclude from the table which methods perform better. Figures 1 to 3 show results for the 5*th*, 50*th* and 95*th* percentiles of true and estimated cost efficiencies. We compare the distribution of each method with the TCE.[14] For visual simplicity, we show only the case when $n = 100$. Figures with sample size $n = 200$ do not vary greatly and are available upon request.

The figures show that results depend on the value of the Λ parameter. As expected, when the variance of the error term increases our results are less accurate (note that non-parametric methodologies assume the absence of noise). In contrast, when the variance of the inefficiency term increases, our results are more precise.

Under **scenario 1** (see Figures 1a, 1c and 1e), when both error and inefficiency terms are relatively small, DEA and KSW methodologies consistently underestimate efficiency (their distributions are below the true efficiency in all percentiles). If we consider median values and density modes, order-*m* tends to overestimate efficiency in all percentiles, while FDH also tends to overestimate efficiency at the 5*th* and 50*th* percentiles. Moreover, we observe that FDH performs well in estimating the efficiency units in the 95*th* percentile.

Although **scenario 4** (see Figures 2b, 2d and 2f) is the opposite case to scenario 1, when both error and inefficiency terms are relatively large they have the same value of Λ. As in scenario 1, DEA and KSW methodologies consistently underestimate efficiency. On the other hand, we see from the 5*th* percentile that both FDH and order-*m* tend to overestimate efficiency. However, at the 50*th* and 95*th* percentiles both methods perform better at estimating the efficiency units since their median values and density modes are closer to the TCE distribution.

Similarly, in **scenario 2** (see Figures 1b, 1d and 1f), when the error term is relatively large but the inefficiency term is relatively small, DEA and KSW tend to underestimate the true efficiency scores, while FDH and order-*m* appear to be close to the TCE distribution (in terms of median values and mode). This scenario yields the poorest results as the dispersion of TCE is much more squeezed than the estimators' distributions. Therefore, when Λ is small, all four methodologies perform less well in predicting efficiency scores.

Scenario 3 (see Figures 2a, 2c and 2e), the error term is relatively small but the inefficiency term is relatively large. Because the Λ value has increased, all methodologies do better at predicting the efficiency scores. At the 5th and 50th percentiles, we observe that DEA and KSW underestimate efficiency, while order-*m* and FDH tend to overestimate it. However, if

[14]We consider that a particular methodology has a better or worse performace depending on the similarities found between its efficiency distribution and the true efficiency distribution.

we consider the median and density modes, DEA (followed by KSW) is closer to the TCE distribution in both percentiles. At the 95th percentile FDH does better at estimating the efficient units, while DEA and KSW slightly underestimate efficiency and order-m slightly overestimates it.

In **scenario 5** (see Figures 2a, 2c and 2e), the error variation is relatively small but the inefficiency variation is very large. This scenario shows the most favourable results because the TCE distribution is highly dispersed and therefore better represents the estimators' performance. At the $5th$ and $50th$ percentiles DEA and KSW densities are very close to the true distribution of efficiency, while FDH and order-m overestimate it. In contrast, at the $95th$ percentile FDH seems to be closer to the TCE although it slightly overestimates it.

Finally, in **scenario 6** (see Figures 3b, 3d and 3f) the error term is relatively large and the inefficiency term is even larger. Again, we observe that when the variation of the inefficiency term increases (compared with scenarios 2 and 4), all the estimators perform better. At the $5th$ and $50th$ percentiles, DEA and KSW slightly underestimate efficiency and FDH and order-m slightly overestimate it (in terms of median values and density mode). However, despite all methods being quite close to the TCE distribution, DEA underestimates less than KSW, and FDH overestimates less than order-m. Finally, at the $95th$ percentile FDH (followed by order-m) is the best method to determine a higher number of efficient units because its mode and median values are closer to the true efficiency.

To sum up, in this subsection we have provided the baseline results for the relative performance of our four non-parametric methodologies. We have considered four median measures as well as other percentiles of the efficiency distribution. We found that the performance of the estimators vary greatly according to each particular scenario. However, we observe that both DEA and KSW consistently underestimate efficiency in nearlly all cases, while FDH and order-m tend to overestimate it. Moreover, we note that DEA and KSW perform best at identifying the ranks of the efficiency scores. In section 4.5 we will explain in greater detail which estimator to use in the various scenarios.

4.4. Robustness checks

We consider a number of robustness checks to verify that our baseline experiment represents the performance of our estimators. Results for each robustness test are given in the extra

Appendix.

- No noise: All our non-parametric estimators assume the absence of noise. However, in the baseline experiment we include noise in each scenario. In this situation, we consider the case where there is no noise in the data generating process. Results show that DEA and KSW perform better at predicting the efficiency scores, while FDH and order-m are slightly worse than the baseline experiment. All methods perform better at estimating the true ranks, except order-m in scenario 1. In short, we find that when noise is absent, DEA and KSW have a greater performance.

- Changes in sample size: The baseline experiment analyses two different sample sizes, n= 100 and 200. We also consider the case where the sample size is very large, that is, n= 500. There is a slight deterioration in the performance of DEA and KSW, while FDH and order-m vary depending on the scenario. However, the results only differed slightly. We find no qualitative changes from the baseline results.

- Returns to scale: The baseline experiment assumes CRS technology. We also consider the case where the technology assumes decreasing and increasing returns to scale ($\gamma = 0.8$ and $\gamma = 1.2$). We find a slight deterioration in the performance of DEA and KSW estimators. Performance for order-m improves with decreasing returns to scale and deteriorates with increasing returns to scale, while FDH varies depending on the scenario. However, despite these minor quantitative differences, the qualitative results do not change.

- Different m values for order-m: Following Daraio and Simar's (2007) suggestion, in order to choose the most reasonable value of m we considered different m sizes ($m = 20$, 30 and 40). In our application the baseline experiment sets $m = 30$. In general, compared with the other m values there are some quantitative changes (i.e., performance with $m = 20$ worsens, while with $m = 40$ it improves slightly); however, the qualitative results from the baseline case seem to hold.

In sort we find that after considering several robustness checks, we do not see any major differences from the baseline experiment. Therefore, despite the initial assumptions done, our simulations accurately depict the performance of our estimators.

4.5. Which estimator in each scenario

Based on the above comparative analysis of the four methodologies' performance, inspired by our results as well as Badunenko et al.'s (2012) proposal, we summarise which ones should be used in the various scenarios, assuming that the simulations remain true for different data generating processes. Table 5 suggests which estimators to use for each scenario when taking into account the efficiency scores. The first row in each scenario shows the relative magnitudes of the estimators compared with the True Cost Efficiency (TCE), while the rest of the rows suggest which estimators to use for each percentile (5*th*, 50*th* or 95*th*). In some cases the methodologies vary little in terms of identifying the efficiency scores.

Badunenko et al. (2012) conclude that if the Λ value is small, as in scenario 2 ($\Lambda = 0.2$), the efficiency scores and ranks will be poorly estimated.[15] This scenario yields the worst results, since the estimators are far from the "truth". Although Table 5 suggests scenario 2, we do not recommend efficiency analysis for this particular scenario, since it would be inaccurate.

Although scenarios 1 and 4 present better results than scenario 2 (when $\Lambda = 1$), estimators also perform poorly at predicting the true efficiency scores. In scenario 1, FDH seems to be the best method to estimate efficiency in all percentiles; however, DEA should also be considered at the 5*th* percentile (the TCE remains between DEA and FDH at this percentile). Similarly, in scenario 4 FDH predominates at the 5*th* percentile, although DEA should also be considered. On the other hand, both FDH and order-*m* perform better at the 50*th* and 95*th* percentiles. For efficiency rankings, DEA and KSW methodologies show a fairly good performance when ranking the observations in both scenarios.

Similarly, scenario 6 performs better than scenarios 1 and 4, since the variation of the inefficiency term increases and, as a consequence, the value of Λ also increases ($\Lambda = 2$). In this scenario the best methodologies for estimating the true efficiency scores seem to be DEA and FDH at the 5*th* and 50*th* percentiles, and FDH (followed by order-*m*) at the 95*th* percentile. In contrast, DEA and KSW methodologies are better at ranking the observations.

In scenario 3, the Λ value increases again ($\Lambda = 5$), and all the methodologies predict the efficiency scores more accurately. For the 5*th* and 50*th* percentiles, the closest estimator to the true efficiency seems to be DEA (followed by KSW). At the 95*th* percentile FDH is the best method. For the rankings, however, DEA and KSW provide more accurate estimations of the

[15]It is difficult to obtain the inefficiency from a relatively large noise component.

efficiency rankings.

Finally, scenario 5 has the largest Λ value ($\Lambda = 10$). Here, the estimators perform best at estimating efficiency and ranks. DEA (followed by KSW) performs better at the $5th$ and $50th$ percentiles and FDH at the $95th$ percentile. DEA and KSW excel at estimating the efficiency rankings.

5. Which estimator performs better with Spanish local governments'

Finally, in this section we identify the most appropriate methodologies to measure local government efficiency in Spain. First, we estimate Λ values for our particular dataset via Fan et al.'s (1996) non-parametric kernel estimator, hereafter FLW.[16] The estimated Λ value helps to determine in which scenario our data lies (see Table 3). Second, we refer to Table 5, check the recommendations for our scenario, and choose the appropriate estimators for our particular needs.

Table 6 reports results of the Λ parameters for our sample of 1,574 Spanish local governments for municipalities between 1,000 and 50,000 inhabitants for the 2008–2013 period. The results of the Λ estimates range from 1.69 to 2.21, which are closer to 2 and correspond to scenario 6. Moreover, the goodness-of-fit measure (R^2) of our empirical data lies at around 0.8. The summary statistics for the overall cost-efficiency results averaged over all municipalities for each year are reported in Table 7. Figure 4 shows the violin plots of the estimated cost efficiencies for further interpretation of results.[17]

In scenario 6, the DEA and FDH methods performed better than the others at the 5^{th} and 50^{th} percentiles of the distribution (the former slightly underestimates efficiency while the latter slightly overestimates it), and FDH (followed by order-m) performed better at the 95^{th} percentile. Therefore, the true efficiency would lie between the results of DEA and FDH both at the median and the lower percentiles, while FDH perform best at estimating the benchmark units. When using these results for performance decisions, local managers must be aware of which part of the observations are of particular interest and whether interest lies in the efficiency score or the ranking. In this context, DEA results indicate that the average

[16] In the appendix we describe how to obtain Λ measures via FLW derived from a cost function.

[17] For visual simplicity, we plot together years 2008–2013, however they do not differ greatly and individual plots are available upon request.

cost efficiency during the period 2008–2013 at the central part of the distribution is 0.54, while the average in FDH is 0.77, so we expect the true cost efficiency scores to lie between 0.54 and 0.77. Moreover, average scores at the lowest quartile (Q1) are 0.42 in DEA and 0.61 in FDH, so we expect the true efficiency scores at the lower end of the distribution to lie between 0.42 and 0.61. Similarly, the average FDH scores at the upper quartile (Q3) are 0.99, so we expect these estimated efficiencies will be similar to the true ones.

The efficiency scores shown by KSW are smaller than those reported by DEA and FDH (the average efficiency scores in KSW for the period 2008–2013 are 0.36 for the lowest quantile (Q1), 0.48 for the mean and 0.57 for the upper quartile (Q3)). Based on our Monte Carlo simulations, we believe that KSW methodology consistently underestimates the true efficiency scores. In contrast, all the statistics estimated by order-*m* methodology are larger than those shown in DEA and FDH (the average efficiency scores in order-*m* for the period 2008–2013 are 0.67 for the lowest quantile (Q1), 0.83 for the mean and 1.00 for the upper quartile (Q3)). Therefore, the experiment leads us to understand that the order-*m* method overestimates the true efficiency scores.

As regards the rank estimates, note that in scenario 6, DEA and KSW methodologies performed best at identifying the ranks of the efficiency scores. Table 8 shows the rank correlation between the average cost efficiency estimates of the four methodologies for the period 2008–2013. As our Monte Carlo experiment showed, DEA and KSW have a high correlation between their rank estimates because of their similar distribution of the rankings. Accordindly, our results show a relatively high correlation between the rank estimates of these two estimators (0.90). Moreover, although there is a relatively high correlation between order-*m* and FDH rank estimates with DEA and KSW, the latter two outperform order-*m* and FDH. As a consequence, DEA and KSW estimators would be preferred to identify the efficiency rankings, but order-*m* and FDH will not necessarily produce poor efficiency rankings.

6. Conclusion

Over the last years, many empirical research studies have set out to evaluate efficiency in local governments. However, despite this high academic interest there is still a lack of a clear, standard methodology to perform efficiency analysis. Since there is no obvious way to choose an estimator, the method chosen may affect the efficiency results, and could provide "unfair" or biased results. In this context, if local regulators take a decision based on an incorrect efficiency score, it could have relevant economic and political implications. Therefore, we note that each methodology leads to different cost efficiency results for each local government, but one method must provide efficiency scores that will be more reliable or closer to the *truth* (Badunenko et al., 2012).

In this setting, the current paper has attempted to compare four different non-parametric estimators: DEA, FDH, order-m and KSW. All these approaches have been widely studied in the previous literature, but little is known about their performance in comparison with each other. Indeed, no study has compared these efficiency estimators. In contrast to previous literature, which has regularly compared techniques and made several proposals for alternative ones, we followed the method applied in Badunenko et al. (2012) to compare the different methods used via Montecarlo simulations and choose the ones which performed better with our particular dataset, in other words, the most appropriate methods to measure local government cost efficiency in Spain.

Our data included 1,574 Spanish local governments between 1,000 and 50,000 inhabitants for the period 2008–2013. Note that the period considered is also important, since the economic and financial crisis that started in 2007 has had a huge impact on most Spanish local government revenues and finances in general. Under these circumstances, identifying a method for evaluating local governments' performance to obtain reliable efficiency scores and set benchmarks over time is even more important, if possible.

In general, we have observed that there is no approach suitable for all efficiency analysis. When using efficiency results for policy decisions, local regulators must be aware of which part of the efficiency distribution is of particular interest (for example, identifying benchmark local governments might be important to decide penalty decisions to poor performers) and if the interest lies in the efficiency scores or the rankings, i.e., it should be considered where and when to use a particular estimator. It is obvious that obtaining reliable efficiency scores might have some implications for local management decisions. Therefore, gaining deeper insights into the issue of local government inefficiency might help to further support effective policy measures, both those that might be appropriate as well as those that are not achieving the their objectives.

We learn that, for our sample of Spanish local governments, all methods showed some room for improvement in terms of possible cost efficiency gains, although some differences in the inefficiency levels obtained were also present. The methodologies which perform better with our sample of Spanish local governments are the DEA and FDH methods at the median and lower tail of the efficiency distribution (the former slightly underestimates efficiency while the latter slightly overestimates it), and FDH (followed by order-m) for local governments with higher performance, according to the findings in our simulations. Specifically, the results suggested that the average true cost efficiency would range between 0.54 and 0.77 during the period 2008–2013, suggesting that Spanish local governments could achieve the same level of local outputs with between 23% and 36% fewer resources. Similarly, the true efficiency scores at the lowest quantile would lie between 0.42 and 0.61, and at the upper quartile would be around 0.99. Further, DEA and KSW methodologies performed best at identifying the ranks of the efficiency scores.

The obtained results provide evidence as to how efficiency could certainly be assessed as close as possible in order to provide some additional guidance for policy makers. In addition, these results are particularly important given the overall financial constraints faced by Spanish local governments during the period under analysis, which have come under increasing pressure to meet strict budgetary and fiscal constraints without reducing their provision of local public services. Therefore, identifying accurately efficiency gains might help to limit the adverse impact of spending cuts on local governments' service provision.

We also note that the effects on the methodological choice identified in this paper might be valid only for our sample dataset. However, the analytical tools introduced in this study could have significant implications for researchers and policy makers who analyse efficiency using data from different countries. From a technical point of view, our results are obtained using a consistent method, which provides a significant contribution to previous literature in local governments efficiency. We emphasize that few studies from this literature have attempted to use two or more alternative approaches in a comparative way (Narbón-Perpiñá and De Witte, 2017a). Therefore, from a policy perspective one should take care when interpreting results and drawing conclusions from these research studies that have used only one particular methodology, since their results might be affected by the approach taken. We think that the implementation of our proposed method to compare different efficiency estimators would represent an interesting contribution that provides the opportunity for further research in this particular issue, given the lack of a clear and standard methodology to perform efficiency analysis.

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Estimation of Λ

We use the following semi-parametric stochastic cost frontier model:

$$C_i = g(y_i) + \varepsilon_i, \qquad i = 1, \ldots.., n, \tag{8}$$

where y_i is a $p \times 1$ vector of random regressors (outputs), $g(.)$ is the unknown smooth function and ε_i is a composed error term, which has two components: (1) v_i, the two-sided random error term which is assumed to be normally distributed N(0, σ_v^2), and (2) u_i, the cost efficiency term which is half-normally distributed ($u_i \geq 0$). These two error components are assumed to be independent.

We use available data on cost (municipal budgets) due to the difficulty of using market prices to measure public services. Hence the assumption allows us to omit the factor prices from the model.

We derive the concentrated log-likelihood function $\ln l(\Lambda)$ and maximise it over the single parameter Λ:

$$\max_{\Lambda} \ln l(\Lambda) = \max_{\Lambda} \left\{ -n \ln \hat{\sigma} + \sum_{i=1}^{n} \ln \left[1 + \Phi\left(\frac{\hat{\epsilon}_i}{\hat{\sigma}} \Lambda \right) \right] - \frac{1}{2\hat{\sigma}^2} \sum_{i=1}^{n} \hat{\epsilon}_i{}^2 \right\}, \tag{9}$$

with $\hat{\epsilon}_i = C_i - \hat{E}(C_i|y_i) + \mu(\hat{\sigma}^2, \Lambda)$ and

$$\sigma^2 = \left\{ \frac{1}{n} \sum_{i=1}^{n} [C_i - \hat{E}(C_i|y_i)]^2 \Big/ \left[1 - \frac{2\Lambda^2}{\pi(1+\Lambda^2)} \right] \right\}^{1/2}, \tag{10}$$

where $\hat{E}(C_i|y_i)$ is the kernel estimator of the conditional expectation $E(C_i|y_i)$ and it is given as:

$$\hat{E}(C_i|y_i) = \sum_{j=1}^{n} C_j \cdot K\left(\frac{y_i - y_j}{h} \right) \Big/ \sum_{j=1}^{n} K\left(\frac{y_i - y_j}{h} \right), \tag{11}$$

where $K(.)$ is the kernel function and $h = h_n$ is the smoothing parameter. For further details about the estimation procedure see Fan et al. (1996).

Table 1: Minimum services provided by all local governments and output variables

	Minimum services	Output indicators
In all municipalities:	Public street lighting	Number of lighting points
	Cemetery	Total population
	Waste collection	Waste collected
	Street cleaning	Street infrastructure surface area
	Supply of drinking water to households	Length of water distribution networks (m)
	Sewage system	Length of sewer networks (m)
	Access to population centres	Street infrastructure surface area
	Paving of public roads	Street infrastructure surface area
	Regulation of food and drink	Total population

Table 2: Descriptive statistics for data in inputs and outputs, period 2008-2013

	Mean	S.d.
INPUTS[a]		
Total costs (X_1)	6,856,864.55	7,990,865.20
OUTPUTS		
Total population (Y_1)	7,555.36	8,460.33
Street infrastructure surface area[b] (Y_2)	336,673.55	325,808.07
Number of lighting points (Y_3)	1,519.78	1,567.02
Tons of waste collected (Y_4)	4,216.73	19,720.07
Length of water distribution networks[b] (Y_5)	50,503.12	93,877.89
Length of sewer networks[b] (Y_6)	29,650.29	32,424.83

[a] In thousands of euros.
[b] In square metres.

Table 3: Combinations of error and inefficiency terms in Monte Carlo simulations to model scenarios

We simulate six different combinations for the error (σ_v) and inefficiency (σ_u) terms, in order to model various real scenarios. The rows represent the variation of the error term, while the columns represent the variation of the inefficiency term. The Λ parameter is the ratio between σ_u and σ_v, which sets each scenario.

	$\sigma_u = 0.01$	$\sigma_u = 0.05$	$\sigma_u = 0.1$
$\sigma_v = 0.01$	s1: $\Lambda = 1.0$	s3: $\Lambda = 5.0$	s5: $\Lambda = 10.0$
$\sigma_v = 0.05$	s2: $\Lambda = 0.2$	s4: $\Lambda = 1.0$	s6: $\Lambda = 2.0$

Table 4: Baseline results with Cobb-Douglas cost function

This table provides the baseline results for the performance of the methodologies in the Monte Carlo experiment. We simulate six scenarios, which represent different combinations for the error (σ_v) and inefficiency (σ_u) terms.

	Bias[a]				**RMSE**[b]				**Upward Bias**[c]				**Kendall's τ**[d]			
	DEA	**FDH**	**Order-m**	**KSW**	**DEA**	**FDH**	**Order-m**	**KSW**	**DEA**	**FDH**	**Order-m**	**KSW**	**DEA**	**FDH**	**Order-m**	**KSW**
s1: $\sigma_v = 0.01$, $\sigma_u = 0.01$																
n=100	-0.0298	0.0074	0.0231	-0.0350	0.0330	0.0096	0.0307	0.0375	0.0500	0.9800	0.9900	0.0100	0.2491	0.1227	0.0615	0.2549
n=200	-0.0348	0.0070	0.0287	-0.0391	0.0376	0.0094	0.0370	0.0415	0.0250	0.9600	0.9850	0.0050	0.2573	0.1590	0.0790	0.2588
s2: $\sigma_v = 0.05$, $\sigma_u = 0.01$																
n=100	-0.0892	-0.0111	0.0049	-0.1006	0.1013	0.0338	0.0399	0.1111	0.0500	0.6400	0.6900	0.0100	0.0681	0.0551	0.0511	0.0687
n=200	-0.1028	-0.0205	0.0043	-0.1130	0.1134	0.0428	0.0466	0.1225	0.0250	0.5000	0.6200	0.0050	0.0707	0.0597	0.0542	0.0705
s3: $\sigma_v = 0.01$, $\sigma_u = 0.05$																
n=100	-0.0182	0.0322	0.0477	-0.0246	0.0238	0.0392	0.0548	0.0285	0.1200	1.0000	1.0000	0.0600	0.6753	0.4443	0.3169	0.6877
n=200	-0.0239	0.0289	0.0512	-0.0293	0.0281	0.0351	0.0581	0.0325	0.0700	0.9900	1.0000	0.0350	0.6843	0.5192	0.3812	0.6911
s4: $\sigma_v = 0.05$, $\sigma_u = 0.05$																
n=100	-0.0707	0.0133	0.0303	-0.0832	0.0857	0.0410	0.0528	0.0960	0.0900	0.7500	0.8100	0.0400	0.3060	0.2547	0.2415	0.3059
n=200	-0.0849	0.0024	0.0279	-0.0963	0.0972	0.0421	0.0565	0.1072	0.0500	0.6300	0.7550	0.0250	0.3132	0.2710	0.2564	0.3123
s5: $\sigma_v = 0.01$, $\sigma_u = 0.1$																
n=100	-0.0101	0.0525	0.0684	-0.0177	0.0203	0.0624	0.0768	0.0238	0.2000	1.0000	1.0000	0.1000	0.8057	0.5928	0.5146	0.8182
n=200	-0.0170	0.0453	0.0689	-0.0232	0.0230	0.0537	0.0763	0.0275	0.1150	0.9950	1.0000	0.0600	0.8174	0.6586	0.5738	0.8254
s6: $\sigma_v = 0.05$, $\sigma_u = 0.1$																
n=100	-0.0580	0.0347	0.0519	-0.0724	0.0755	0.0591	0.0722	0.0869	0.1300	0.8200	0.8800	0.0700	0.4996	0.4170	0.4030	0.4990
n=200	-0.0726	0.0207	0.0485	-0.0854	0.0867	0.0530	0.0717	0.0974	0.0750	0.7200	0.8400	0.0400	0.5065	0.4403	0.4332	0.5065

[a] The bias reports the difference between the estimated and true efficiency scores. When it is negative (positive), the estimators are underestimating (overestimating) the true efficiency.
[b] The $RMSE$ (root mean squared error) measures the standard deviation or error from the true efficiency.
[c] The upward bias is the proportion of estimated efficiencies larger than the true efficiencies (returning a value of 1). The desired value is 0.5. The values less (greater) than 0.5 indicate underestimation (overestimation) of cost efficiencies.
[d] Kendall's τ shows the correlation coefficient for the efficiency ranks between true and estimated efficiency scores.

Table 5: Relative performance of the efficiency scores at the $5th$, $50th$ and $95th$ percentiles

This table summarises which estimators to use for each scenario. Each scenario represent different combinations for the error (σ_v) and inefficiency (σ_u) terms. The first row of each scenario represents the relative magnitudes of the estimators compared with the True Cost Efficiency (TCE), while the rest of the rows suggest which estimators to use for each percentile ($5th$, $50th$ or $95th$).

	σ_u**=0.01**	σ_u**=0.05**	σ_u**=0.1**
σ_v**=0.01**	**scenario 1:** **KSW<DEA<TCE<FDH<order-*m*** 5: DEA or FDH 50: FDH 95: FDH	**scenario 3:** **KSW<DEA<TCE<FDH<order-*m*** 5: DEA 50: DEA 95: FDH	**scenario 5:** **KSW<DEA<TCE<FDH<order-*m*** 5: DEA 50: DEA 95: FDH
σ_v**=0.05**	**scenario 2:** **KSW<DEA<FDH<TCE<order-*m*** 5: order-*m* or FDH 50: order-*m* or FDH 95: order-*m* or FDH	**scenario 4:** **KSW<DEA<TCE<FDH<order-*m*** 5: DEA or FDH 50: order-*m* or FDH 95: order-*m* or FDH	**scenario 6:** **KSW<DEA<TCE<FDH<order-*m*** 5: DEA or FDH 50: DEA or FDH 95: order-*m* or FDH

Table 6: Estimates to determine the scenario for Spanish local governments dataset

This table contains the results of the Λ parameters of our sample of 1,574 Spanish local governments for the period 2008–2013. Λ values help to determine in which scenario our data lies. We also report the goodness-of-fit measure (R^2) of our empirical data.

	2008	**2009**	**2010**	**2011**	**2012**	**2013**
Λ	2.0596	2.2143	1.7256	1.6953	1.8283	1.8371
R^2	0.7980	0.8331	0.8250	0.8244	0.8209	0.8478

Table 7: Summary statistics for efficiency results in Spanish local governments

	DEA					
	Mean	**Median**	**Min**	**Max**	**Q1**	**Q3**
2008	0.4943	0.4689	0.0437	1.0000	0.3611	0.6038
2009	0.5843	0.5740	0.1257	1.0000	0.4633	0.6830
2010	0.5212	0.4953	0.1312	1.0000	0.4017	0.6135
2011	0.5314	0.5092	0.1359	1.0000	0.4104	0.6237
2012	0.5316	0.5128	0.1079	1.0000	0.4077	0.6429
2013	0.5712	0.5591	0.1138	1.0000	0.4458	0.6823
2008–2013	0.5390	0.5199	0.1097	1.0000	0.1761	0.6415

	FDH					
	Mean	**Median**	**Min**	**Max**	**Q1**	**Q3**
2008	0.7444	0.7678	0.0808	1.0000	0.5644	1.0000
2009	0.8186	0.8563	0.2045	1.0000	0.6821	1.0000
2010	0.7761	0.7848	0.1559	1.0000	0.6251	1.0000
2011	0.7453	0.7434	0.2037	1.0000	0.5808	0.9892
2012	0.7630	0.7737	0.1497	1.0000	0.6104	1.0000
2013	0.7619	0.7721	0.1497	1.0000	0.6104	0.9999
2008–2013	0.7682	0.7830	0.1574	1.0000	0.2053	0.9982

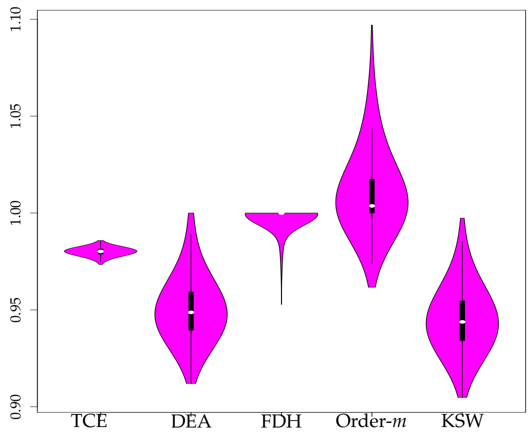
	Order-m					
	Mean	**Median**	**Min**	**Max**	**Q1**	**Q3**
2008	0.8089	0.8255	0.0834	1.9813	0.6312	1.0000
2009	0.8691	0.8926	0.2122	1.7369	0.7318	1.0013
2010	0.8385	0.8515	0.2172	1.8080	0.6938	1.0000
2011	0.8088	0.8100	0.2368	2.0281	0.6497	1.0000
2012	0.8222	0.8358	0.1797	1.8914	0.6644	1.0000
2013	0.8209	0.8328	0.1785	1.9204	0.6609	1.0000
2008–2013	0.8281	0.8414	0.1846	1.8944	0.6720	1.0002

	KSW					
	Mean	**Median**	**Min**	**Max**	**Q1**	**Q3**
2008	0.4421	0.4239	0.0400	1.0000	0.3183	0.5454
2009	0.5384	0.5297	0.1179	1.0000	0.4250	0.6370
2010	0.4541	0.4294	0.0563	1.0000	0.3420	0.5399
2011	0.4752	0.4558	0.1178	1.0000	0.3697	0.5558
2012	0.4677	0.4477	0.0134	1.0000	0.3503	0.5687
2013	0.4846	0.4711	0.0118	1.0000	0.3678	0.5848
2008–2013	0.4770	0.4596	0.0595	1.0000	0.3622	0.5719

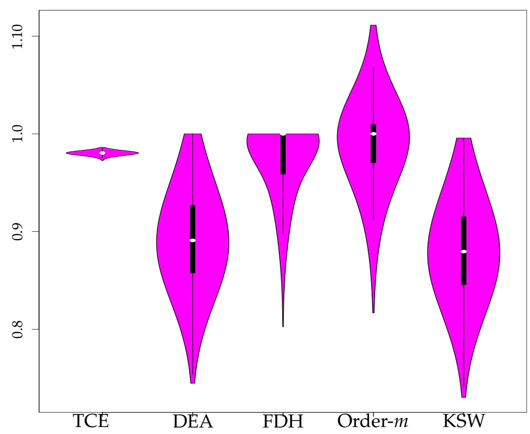
Table 8: Rank correlation Kendall coefficients between the average cost efficiency estimates of the four methodologies for the period 2008–2013

	DEA	**FDH**	**Order-m**	**KSW**
DEA	1.0000	0.6687	0.6463	0.9004
FDH	0.6687	1.0000	0.7755	0.6136
Order-m	0.6463	0.7755	1.0000	0.5801
KSW	0.9004	0.6136	0.5801	1.0000

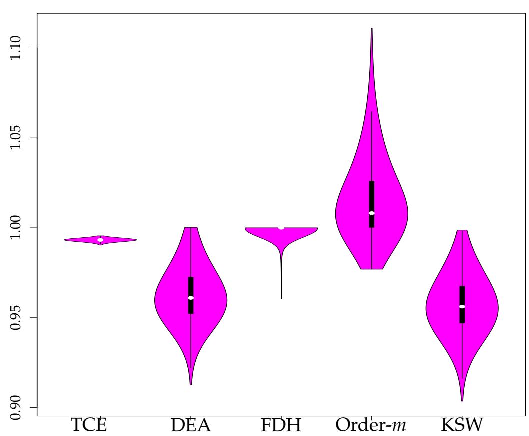
Figure 1: Violin plots in scenario 1 and 2 for 5*th*, 50*th* and 95*th* percentiles of cost efficiency estimates under a Cobb-Douglas cost function.



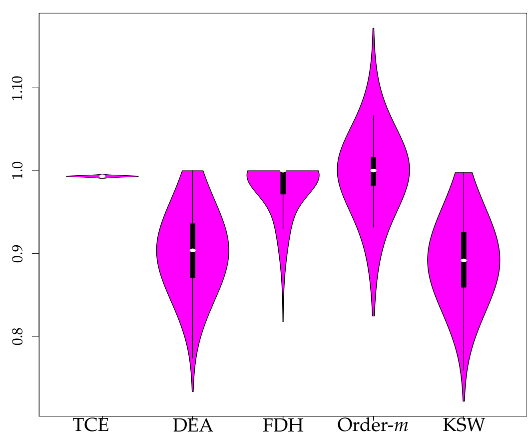
(a) S1, 5th percentile, n=100



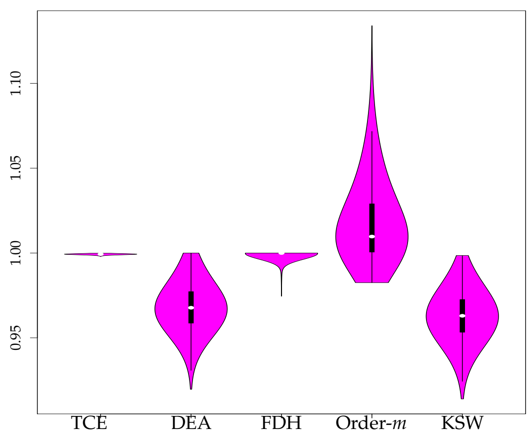
(b) S2, 5th percentile, n=100



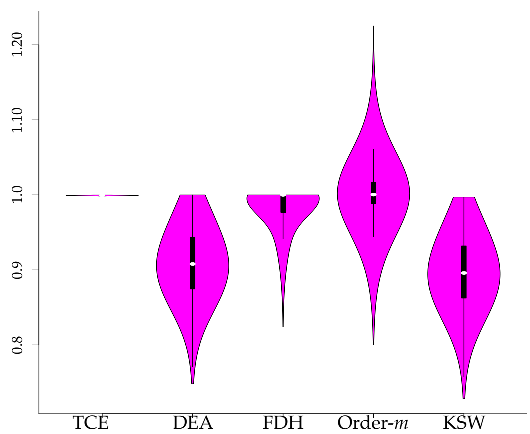
(c) S1, 50th percentile, n=100



(d) S2, 50th percentile, n=100

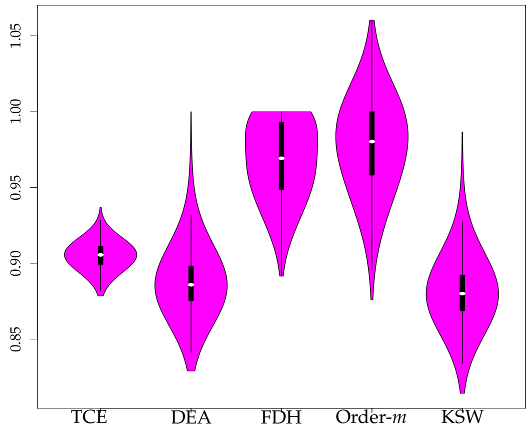


(e) S1, 95th percentile, n=100

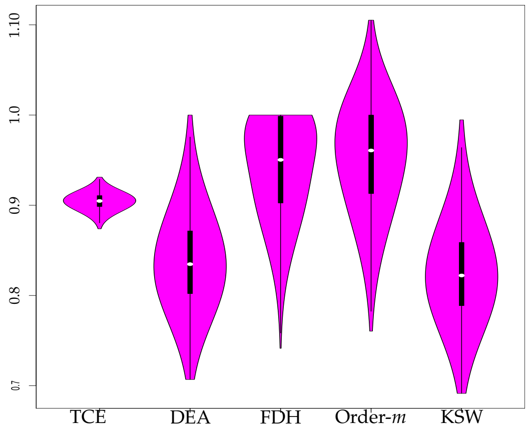


(f) S2, 95th percentile, n=100

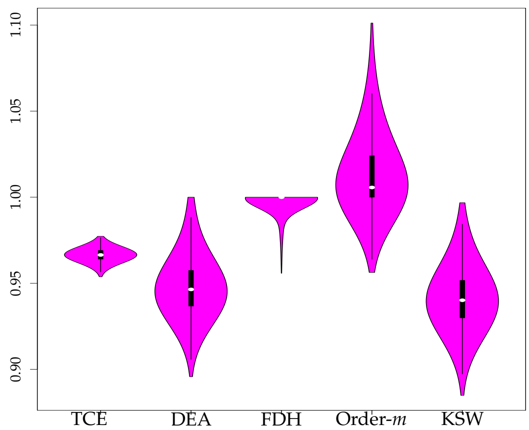
Figure 2: Violin plots in scenario 3 and 4 for 5th, 50th and 95th percentiles of cost efficiency estimates under a Cobb-Douglas cost function.



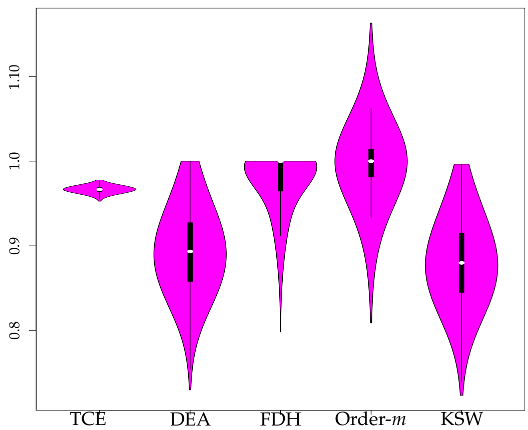
(a) S3, 5th percentile, n=100



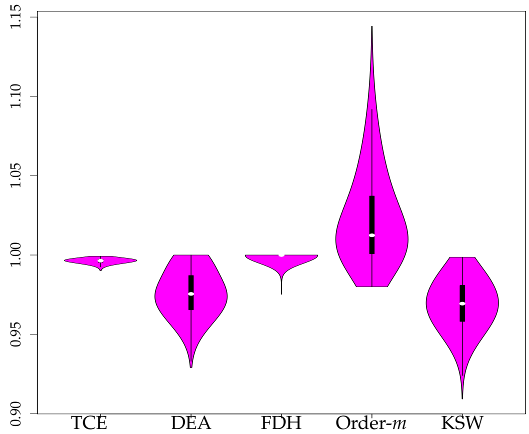
(b) S4, 5th percentile, n=100



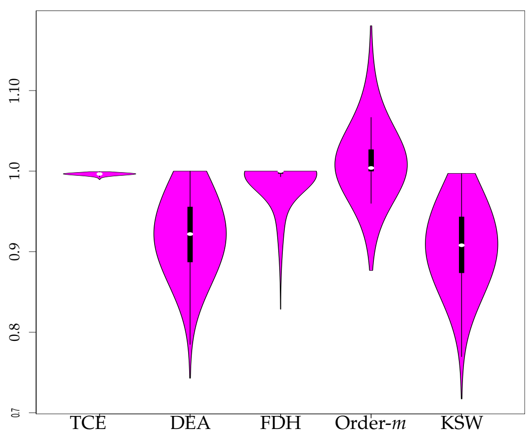
(c) S3, 50th percentile, n=100



(d) S4, 50th percentile, n=100

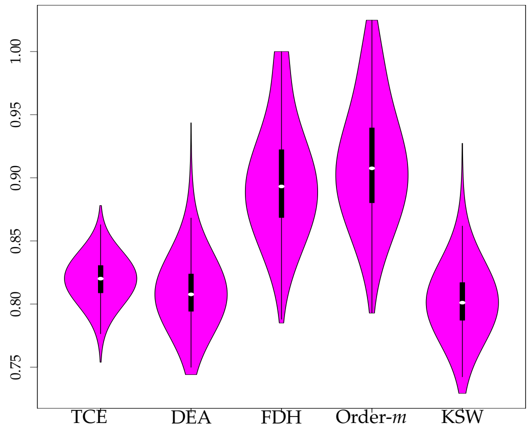


(e) S3, 95th percentile, n=100

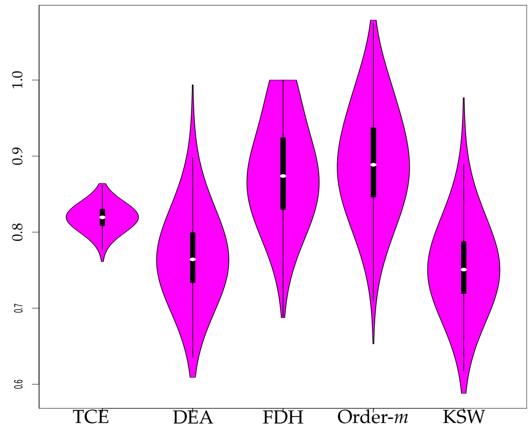


(f) S4, 95th percentile, n=100

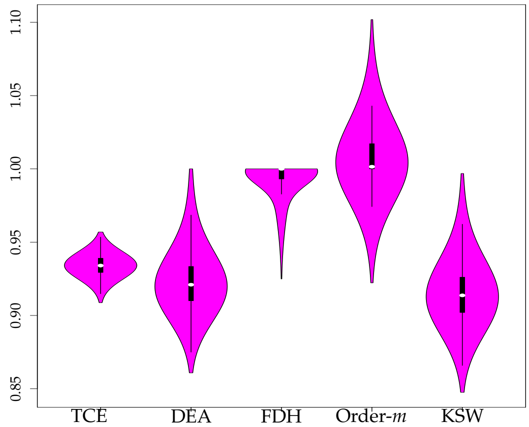
Figure 3: Violin plots in scenario 5 and 6 for 5*th*, 50*th* and 95*th* percentiles of cost efficiency estimates under a Cobb-Douglas cost function.



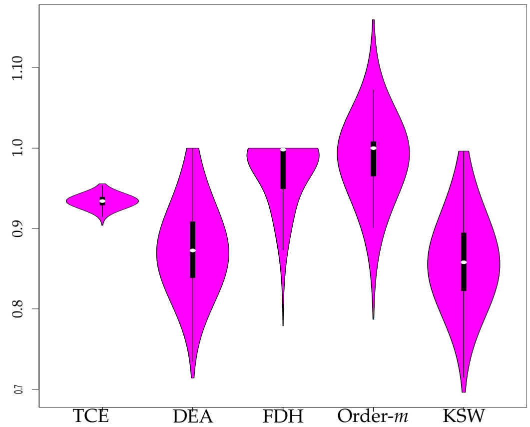
(a) S5, 5th percentile, n=100



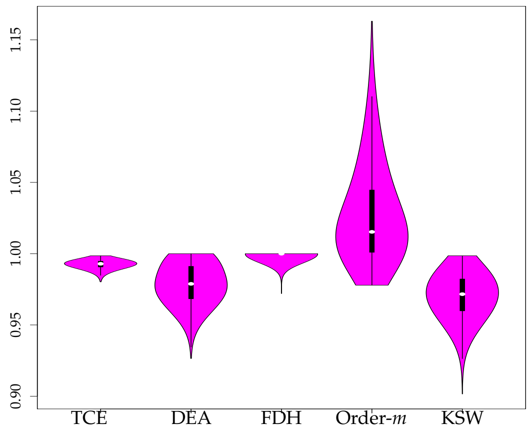
(b) S6, 5th percentile, n=100



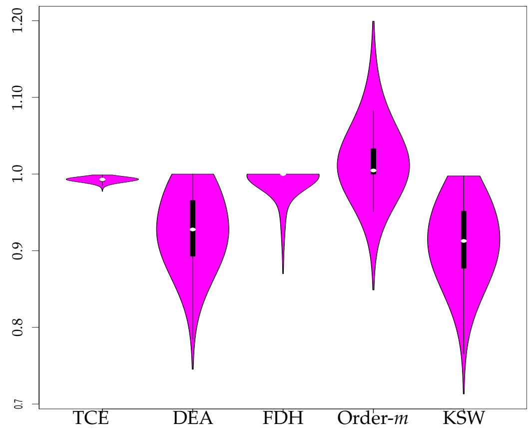
(c) S5, 50th percentile, n=100



(d) S6, 50th percentile, n=100



(e) S5, 95th percentile, n=100



(f) S6, 95th percentile, n=100

Figure 4: Violin plots of cost efficiency estimates in Spanish local governments

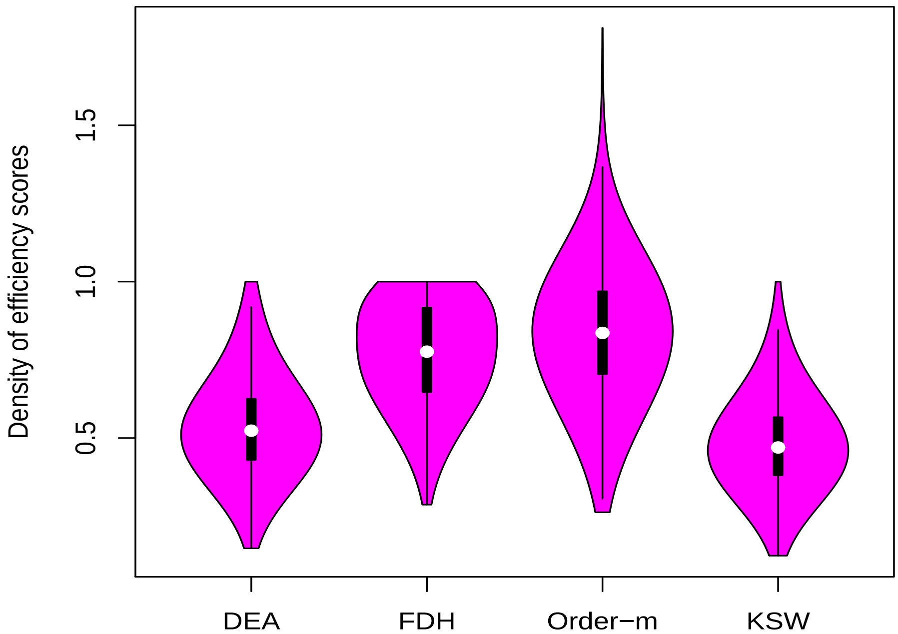


Figure 5: 2008–2013