

Gibrat's law redux: Think profitability
instead of growth

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Abstract

The basic philosophy behind Gibrat's rule of proportionate effect has been to find some common mechanism in the growth process of business firms, based on the idea that growth rates are independent of size and drawn from the same distribution. After decades of research, however, it seems fair to say that the "law" fails to provide a universal mechanism for the growth of firms. Here we take the position that it is more plausible for Gibrat's approach to apply to firm profitability rather than firm growth, in line with the classical idea of economic competition as a dynamic process of capital reallocation. Considering a sample of more than five hundred long-lived US corporations from virtually all sectors, we compare the statistical properties of growth and profit rates over a time span of thirty years, and find that profit rates and their volatilities are independent of size, which is not true of growth rates. We also find that the empirical densities of both profitability and growth can be described by exponential power (or Subbotin) distributions, but there are pronounced differences in their parameterizations and autocorrelation structures. We argue that a recently proposed diffusion process not only reproduces the cross-sectional distribution of profit rates, but is also consistent with the empirical time series of individual firms and their autocorrelations. In the natural sciences such a situation is commonly referred to as a statistical equilibrium, while econometricians speak of ergodicity and stationarity. Our economic interpretation of this property is that all surviving firms are subject to the same competitive pressures of capital reallocation, irrespective of their industry or particular line of business. They all face the same profitability benchmark and volatility, while their idiosyncratic efforts merely have an effect on the persistence of abnormal profits. In other words, survivors have to participate in the same game and can only choose to do so at different "speeds." We conclude with the empirical observation that the speed of convergence from abnormal profits to the system-wide average depends negatively on firm size, diversification, and capital intensity.

Keywords: Profit rates, diffusion process, statistical equilibrium, dynamic competition, persistence

JEL classification: C16, L10, D21, E10

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The basic philosophy behind Gibrat's rule of proportionate effect has been to find some common mechanism in the growth process of business firms, based on the idea that growth rates are independent of size and drawn from the same distribution. After decades of research, however, it seems fair to say that the "law" fails to provide a universal mechanism for the growth of firms. Here we take the position that it is more plausible for Gibrat's approach to apply to firm profitability rather than firm growth, in line with the classical idea of economic competition as a dynamic process of capital reallocation. Considering a sample of more than five hundred long-lived US corporations from virtually all sectors, we compare the statistical properties of growth and profit rates over a time span of thirty years, and find that profit rates and their volatilities are independent of size, which is not true of growth rates. We also find that the empirical densities of both profitability and growth can be described by exponential power (or Subbotin) distributions, but there are pronounced differences in their parameterizations and autocorrelation structures. We argue that a recently proposed diffusion process not only reproduces the cross-sectional distribution of profit rates, but is also consistent with the empirical time series of individual firms and their autocorrelations. In the natural sciences such a situation is commonly referred to as a *statistical equilibrium*, while econometricians speak of ergodicity and stationarity. Our economic interpretation of this property is that all surviving firms are subject to the same competitive pressures of capital reallocation, irrespective of their industry or particular line of business. They all face the same profitability benchmark and volatility, while their idiosyncratic efforts merely have an effect on the persistence of abnormal profits. In other words, survivors have to participate in the same game and can only choose to do so at different "speeds". We conclude with the empirical observation that the speed of convergence from abnormal profits to the system-wide average depends negatively on firm size, diversification, and capital intensity.

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1 Introduction

Our main argument is that the profitability of surviving corporations exhibits remarkable regularity across space and time, and can be conveniently characterized by one and the same diffusion process for all firms. We show that the process is not only consistent with the cross-sectional distribution of profit rates across firms at a given point in time, but is also a reasonable description of the time evolution of profit rates for each surviving firm. Thus we refer to the profitability of surviving corporations as a *statistical equilibrium* outcome.¹ The implications of statistical equilibrium for our understanding of individual firm destinies are stark and unexpected, since the model and data suggest that idiosyncratic firm characteristics are not influencing the aggregate distributional outcome. Instead, idiosyncratic efforts merely seem to have an impact on the individual persistence of abnormal profits from the system-wide average, that is their *speed of adjustment* or *convergence*. We find that large, capital intensive, or broadly diversified survivors exhibit the slowest speed of adjustment across all non-banking sectors of the economy.

The idea that market economies are driven by the reallocation of capital in search of profit rate equalization dates back to classical economics, and is certainly one of the most widely accepted theories of capitalism (see, e.g., Foley, 2006). Traditionally, however, the literature on industrial dynamics has not focused on profitability, but rather on firm size and growth. Starting with Gibrat (1931), the most influential idea to explain the dynamics of individual firms was to claim that firm growth rates should be independent of firm size and usually also of each other.² According to the central limit theorem, this would imply normally distributed growth rates, and a log-normal distribution of firm sizes. Subsequent empirical analyses have shown that size, age, and the life-cycle of firms influence the growth performance in ambiguous ways (see, for instance, the surveys by Santarelli et al., 2006; Sutton, 1997), while the distributional implications of Gibrat’s rule, in particular with respect to the unconditional growth rate distribution, have received less attention until recently.³ The empirical density of growth rates is not Gaussian, however, and therefore at odds with Gibrat’s rule. The non-normality of the growth rate distribution is most likely an imprint of the complex interactions and interdependencies among firms, and would indicate that the independence assumption is violated. Instead, the unconditional distribution of growth rates approximates a double-exponential (or Laplace) distribution, for instance for US manufacturing companies (Stanley et al., 1996), the world’s largest pharmaceutical firms (Bottazzi et al., 2001), and Italian manufacturing sectors (Bottazzi et al., 2002), even at higher levels of sectoral disaggregation (Bottazzi and Secchi, 2006).

Our dataset suggests that the growth rates of long-lived US non-banking corporations are even more leptokurtic than the Laplace distribution, which has recently also been observed in French manufacturing (see Bottazzi et al., 2011). The rate of profit, on the other hand, seems to be much closer to the Laplace distribution than the corresponding growth rates in our sample. At the same time,

¹Foley (1994) and Garibaldi and Scalas (2010) provide useful background reading for readers who might not be entirely familiar with the concept of statistical equilibrium.

²To be more precise, growth rates are iid random variables with finite variance.

³The distribution of firm sizes, on the other hand, has received more attention, and dates back to the work of Simon (see, e.g., Ijiri and Simon, 1977; Simon and Bonini, 1958). More recent findings are essentially concerned with the question whether the size distribution is log-normal (see, e.g., Hart and Oulton, 1996; Stanley et al., 1995) or power-law (see, e.g., Axtell, 2001).

it is not clear how or whether the size of corporations might influence the rate of profit. While Baumol (1967) argues that size could confer market power and lead to abnormally high profit rates, the empirical findings generally do not support his claim (see, e.g., Alexander, 1949; Amato and Wilder, 1985; Goddard et al., 2005; Hall and Weiss, 1967; Marcus, 1969; Whittington, 1980), and we also find that the profitability of surviving US corporations is largely independent of size, in line with classical economic thinking.

A crucial aspect of Gibrat’s model concerns the autocorrelation structure of firm growth rates. Identification of serially dependent growth rates would speak against Gibrat’s hypothesis of random walk growth in firm size, but unfortunately empirical studies of the autocorrelation structure of growth rates yield inconclusive results: positive autocorrelations in firm growth rates are reported by Chesher (1979) and Geroski et al. (1997) for the UK, and by Weiss (1998) for Austria, while Boeri and Cramer (1992) and Goddard et al. (2002) observe negative serial correlations in German and Japanese data. Other studies do not find any significant autocorrelations in firm growth rates (see, for instance, Almus and Nerlinger, 2000; Geroski and Mazzucato, 2002; Lotti et al., 2003), whereas Coad (2007) reports that smaller French manufacturing firms exhibit negatively correlated growth rates, while larger firms display positive autocorrelations. Our results regarding long-lived US corporations indicate that there are no statistically significant autocorrelations in firm growth rates. In contrast, profit rates do exhibit significantly positive autocorrelations in our sample.

The empirical distribution of profit rates has previously been considered by Alfarano et al. (2012), who propose a diffusion model to account for the cross-sectional Laplace distribution of profit rates. Their model will guide our present investigation, and relies on three parameters: a system-wide average rate of profit, a system-wide dispersion measure of profit rates, and an idiosyncratic noise factor that determines the persistence of abnormal profits for individual firms. The model turns out to be consistent with the observed autocorrelation structure of profit rates, and the model’s assumption of a common location and dispersion parameter across all firms is reflected in the data as well. This strongly suggests that the process is a useful description of the time evolution of individual firm profitability. Closed-form solutions for the transient density and the autocorrelation function of the diffusion process enable us to estimate the idiosyncratic noise levels with maximum likelihood, and to compute the adjustment speed from abnormal profits for each firm.

2 Data

The data for this study are taken from Thomson Reuters’ *Datastream* and consist of annual observations for the sales, operating income, total assets, number of employees, and market value of publicly traded US companies. According to the database, a total of 6,860 firms have been present in the market for at least one year from 1980–2011, and have operated in at least one of the 78 different sectors listed in Table 1. Unlike many previous studies that typically focus on the manufacturing sectors (SIC codes 20 to 39), our present analysis considers a diverse set of firms across the different sectors, and merely excludes banks (SIC codes 60 and 61) because their balance sheets exceed those in other sectors by at least an order of magnitude.

Table 1: Sector definitions and number of firms in each sector. Firms operating in more than one sector are classified according to the business segment that generated the most revenue. The fourth column refers to the whole dataset, while the fifth column represents long-lived firms.

Division	SIC	Sector	No. of firms	No. of long-lived firms
Agriculture, forestry, and fishing	01	Agricultural production - crops	20	1
	02	Agricultural production - livestock and animal specialties	3	0
	07	Agricultural services	4	0
	08	Forestry	3	0
	09	Fishing, hunting and trapping	2	0
Mining	10	Metal mining	139	4
	12	Coal mining	27	1
	13	Oil and gas extraction	336	15
	14	Mining and quarrying of nonmetallic minerals, except fuels	19	1
Construction	15	Building construction - general contractors & operative builders	37	5
	16	Heavy construction, except building construction - contractors	26	3
	17	Special trade contractors	21	1
Manufacturing	20	Food and kindred products	141	20
	21	Tobacco products	8	2
	22	Textile mill products	19	2
	23	Apparel, finished products from fabrics & similar materials	35	4
	24	Lumber and wood products, except furniture	25	5
	25	Furniture and fixtures	27	8
	26	Paper and allied products	36	8
	27	Printing, publishing and allied industries	71	15
	28	Chemicals and allied products	636	36
	29	Petroleum refining and related industries	31	10
	30	Rubber and miscellaneous plastic products	55	6
	31	Leather and leather products	19	3
	32	Stone, clay, glass and concrete products	30	6
	33	Primary metal industries	70	9
	34	Fabricated metal products, except machinery & transport equipment	79	15
	35	Industrial and commercial machinery and computer equipment	297	36
	36	Electronic, electronical equipment & components, except computer equipment	555	33
	37	Transportation equipment	147	24
	38	Measuring, analyzing and controlling instruments	422	33
	39	Miscellaneous manufacturing industries	59	4
Transportation, communications, electric, gas, and sanitary services	40	Railroad transportation	9	5
	41	Local, suburban transit and interurban highway passenger transportation	3	0
	42	Motor freight transportation	30	3
	44	Water transportation	41	2
	45	Transportation by air	34	4
	46	Pipelines, except natural gas	10	0
	47	Transportation services	24	0
	48	Communications	227	8
	49	Electric, gas, and sanitary services	191	62
Wholesale trade	50	Wholesale trade-durable goods	145	10
	51	Wholesale trade-nondurable goods	100	11
Retail trade	52	Building materials, hardware, garden supply and mobile home dealers	8	1
	53	General merchandise stores	29	7
	54	Food stores	25	5
	55	Automotive dealers and gasoline service stations	29	1
	56	Apparel and accessory stores	62	7
	57	Home furniture, furnishings, and equipment stores	28	3
	58	Eating and drinking places	85	7
	59	Miscellaneous retail	92	3
Finance, insurance and real estate	62	Security and commodity brokers, dealers, exchanges, and services	72	1
	63	Insurance carriers	141	20
	64	Insurance agents, brokers, and service	19	3
	65	Real estate	85	1
	67	Holding and other investment offices	334	11
Services	70	Hotels, rooming houses, camps, and other lodging places	42	1
	72	Personal services	15	4
	73	Business services	1018	19
	75	Automotive repair, services, and parking	12	1
	76	Miscellaneous repair services	3	1
	78	Motion pictures	54	0
	79	Amusement and recreation services	68	1
	80	Health services	118	5
	81	Legal services	2	0
	82	Educational services	49	1
	83	Social services	5	0
	84	Museums, art galleries, and botanical and zoological gardens	1	0
	86	Membership organizations	1	0
	87	Engineering, accounting, research, management, and related services	187	4
	89	Services, not elsewhere classified	27	0
Public administration	92	Justice, public order, and safety	1	0
	95	Administration of environmental quality and housing programs	1	0
	97	National security and international affairs	8	0
	99	Nonclassifiable establishments	26	0
Total			6860	522

We focus on long-lived or “surviving” firms that we define as companies operating in the market for the entire time span from 1980 to 2011. This panel contains 522 companies that account for more than seventy percent of market capitalization, total assets and employment in the sample, and are therefore a major determinant of macroeconomic activity. The importance of such a ‘granular’ view of the economy has recently been forcefully argued by Gabaix (2011), who finds that about one third of variations in GDP growth in the United States can be attributed to the idiosyncratic destinies of the largest one hundred corporations in the US, and the contribution of the firms in our sample must therefore be considerably larger.⁴

For each company, we compute annual (logarithmic) growth rates g for the different measures of firm size S ,

$$g_{i,t} = \log(S_{i,t+1}) - \log(S_{i,t}), \quad (1)$$

where i runs over firms and t denotes time. We consider sales, total assets, number of employees, and market value as proxies for firm size. Our proxy for the profit rate p is the *return on assets*,

$$p_{i,t} = \frac{I_{i,t}}{A_{i,t}}, \quad (2)$$

where I denotes operating income, and A denotes total assets.

2.1 Descriptive statistics

To understand the distributional properties of growth and profit rates, it is instructive to consider the time evolution of the first four centralized moments of these two quantities, shown in Figure 1. It is noteworthy that the mean and standard deviation of profit rates are relatively stable compared to their growth rate counterparts, indicating that the distribution of profit rates remains stable during the period 1980-2011. The stability of the average profit rate becomes most apparent when we look at the dot-com bubble and the recent financial crisis. During those years a massive drop in market demand was reflected in sizable decreases in firm growth and sometimes even in firm size, with the most extreme fluctuations occurring in the financial market, that is in the growth of market value. The adverse effects on firm profitability, however, appear very moderate in comparison to the growth rate series.⁵ Although Figure 1 merely shows the average behavior, we do observe a decline in growth rates that is more pronounced than the decline in profitability.⁶

As one might expect, the growth rate of market value turns out to be the most volatile quantity. On average, its standard deviation exceeds the standard deviation of profit rates by a factor greater than three. Employment growth appears more volatile than growth in sales or total assets, yet the latter are still approximately twice as volatile as profit rates. The third moments fluctuate around zero, indicating that neither the distribution of growth nor profit rates is systemically skewed, in line

⁴This is particularly true in light of the power law distribution of firm sizes. We will address concerns regarding a potential ‘survivorship bias’ in the final section.

⁵One could speculate that adverse demand shocks induce firms to reduce costs (number of employees) or the scope of their operations (total assets), thereby mitigating the effects of decreasing sales on profitability.

⁶This non-trivial stability of the profit rate over time has also been pointed out by Mundt et al. (2013), who analyze data from more than 30,000 publicly traded firms in more than forty countries that account for about ninety percent of world GDP. Therefore we would like to think that our present findings do not just reflect a peculiarity of the US data.

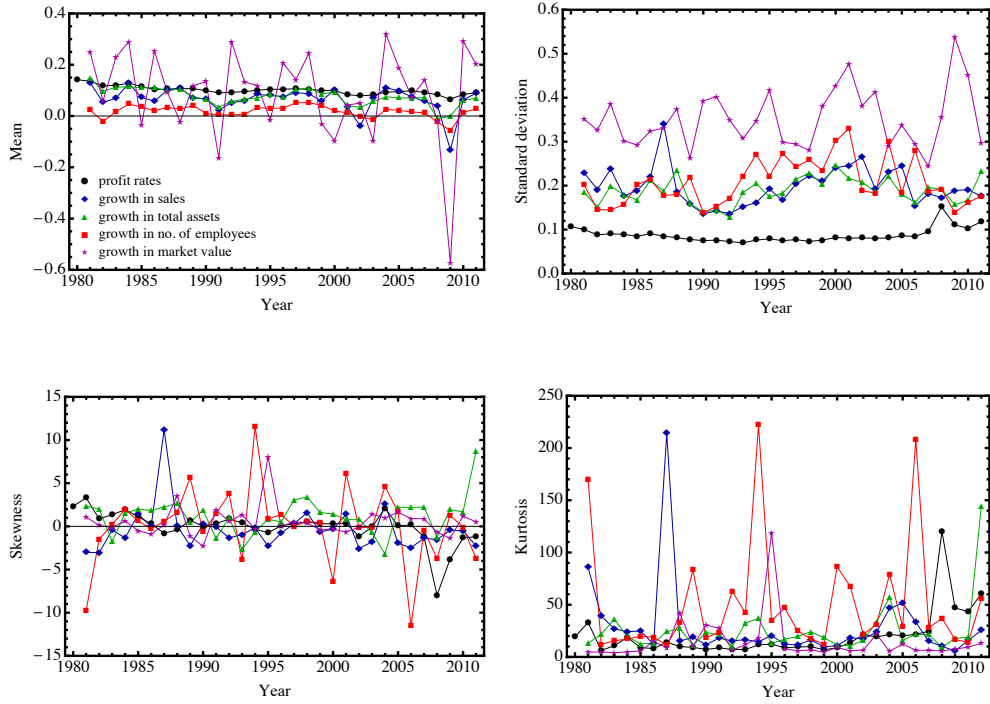


Figure 1: Time evolution of the first four centralized moments of profit rates and the different measures of firm growth rates. Lines between data points are a visual aid.

with previous empirical studies focusing on the profit or growth rates of long-lived firms (see, for instance, Alfarano et al., 2012; Bottazzi and Secchi, 2003; Stanley et al., 1996). The time evolution of the fourth moments indicates fat tails, yet excess kurtosis seems to be more pronounced for growth than for profit rates, in particular growth in number of employees and sales.⁷ The first four moments still provide less information than the distributions of growth and profit rates, so we examine their empirical densities next.

2.2 Empirical densities

In the recent literature on growth rate distributions, it is common practice to eliminate possible trends in firm size by considering the normalized (logarithmic) size

$$s_{i,t} = \log(S_{i,t}) - N^{-1} \sum_{i=1}^N \log(S_{i,t}), \quad (3)$$

which is obtained by subtracting the average (log) size of all long-lived firms from the (log) size of company i . Then the normalized growth rate is defined as the first difference of (3)

$$\tilde{g}_{i,t} = s_{i,t+1} - s_{i,t}. \quad (4)$$

⁷It turns out that the most extreme kurtosis realizations in 1981, 1987, 1994 and 2006 originate from the firms Vornado Realty Trust, Public Storage REIT, Harbinger Group, all with two-digit SIC code 67 (holding and other investment offices), and Arabian American Development (petroleum refining, SIC code 29).

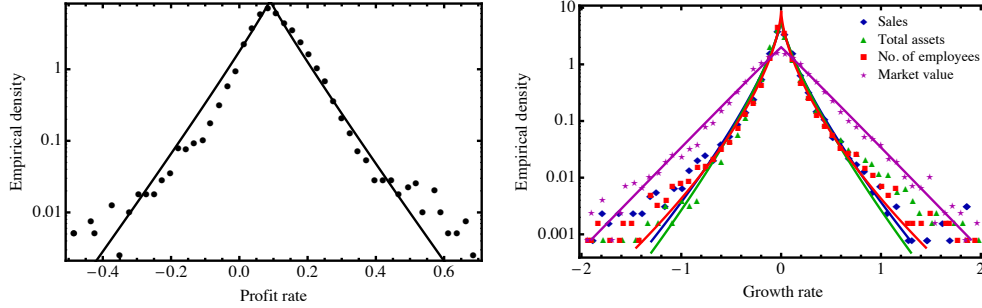


Figure 2: Pooled empirical densities of annual profit rates (left) and growth rates (right) for 522 publicly traded non-bank companies in the United States during the period 1980-2011. The mode of the profit rate distribution is $\hat{m} = 9.3\%$. Due to the normalization of the data, all growth rate distributions are centered around zero. The solid curves show the Subbotin fit obtained by maximum likelihood estimation of the parameters α and σ . Note the log-scale on the vertical axes.

Profit rates, on the other hand, are not normalized in any way and simply remain in the raw form (2). In order to fit the empirical distributions of growth and profit rates in our sample, we follow standard procedure in the field and employ the *exponential power distribution* first suggested by Subbotin (1923). Its functional form reads

$$f(x) = \frac{1}{2\sigma\alpha^{1/\alpha}\Gamma(1+1/\alpha)} \exp\left(-\frac{1}{\alpha} \left|\frac{x-m}{\sigma}\right|^\alpha\right), \quad (5)$$

where $\alpha, \sigma \in \mathbb{R}^+$, $m \in \mathbb{R}$, and $\Gamma(\cdot)$ denotes the Gamma function. The Subbotin is characterized by three parameters: a location parameter m , a scale parameter σ , and a shape parameter α that is responsible for qualitative differences in the distribution, in particular its kurtosis. It is readily verified that the Subbotin density includes the Laplacian ($\alpha = 1$) and the Gaussian ($\alpha = 2$) as special cases.

Figure 2 presents the pooled empirical densities of profit rates and normalized growth rates, as well as the corresponding Subbotin fit obtained from maximum likelihood estimation of the parameters, reported in Table 2. The parameter estimates of the pooled empirical distribution of profit rates are denoted by $\hat{\alpha}$, $\hat{\sigma}$ and $\hat{m}$. We find that the empirical densities of profit and growth rates are clearly non-Gaussian, and can be reasonably well approximated by a symmetric Subbotin distribution. The empirical density of profit rates exhibits a “linear tent-shape” on a semi-log scale that is characteristic of the Laplace distribution. Except for market value growth, the various growth rate distributions are more leptokurtic than the

Table 2: Maximum likelihood estimates of the Subbotin parameters α and σ . Standard errors are shown in parentheses.

	shape parameter $\hat{\alpha}$	scale parameter $\hat{\sigma}$
profit rate	0.95 (0.01)	0.0570 (0.0005)
sales growth rate	0.74 (0.01)	0.0999 (0.0011)
total assets growth rate	0.76 (0.01)	0.0977 (0.0010)
employment growth rate	0.62 (0.01)	0.0806 (0.0009)
market value growth rate	1.01 (0.01)	0.2467 (0.0024)

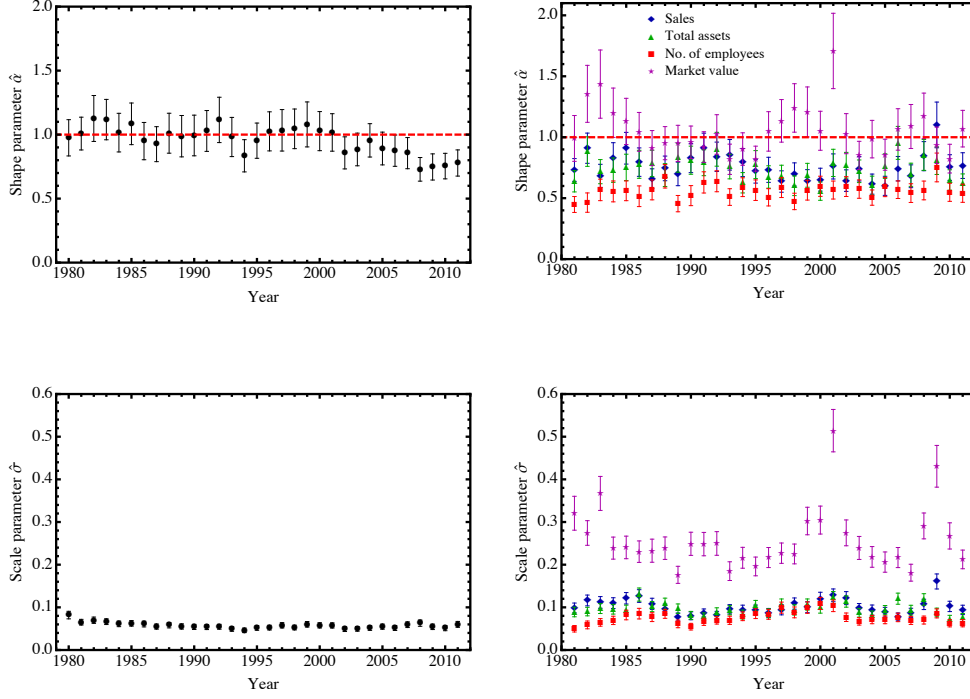


Figure 3: Year-by-year maximum likelihood estimates of the Subbotin shape and scale parameters for profit (left panel) and growth rates (right panel). Error bars show two standard errors.

Laplace.⁸ The visual impression is confirmed by the estimates of the shape parameter α , which are significantly smaller than unity for the growth of sales, total assets and employment.⁹ In spite of large fluctuations in the mean and standard deviation, the closest Laplace fit is obtained for growth rates of market value. Finally, the estimates for the scale parameter confirm that growth rates (in particular for market value) are more volatile than profit rates, while the growth of employment is the most leptokurtic distribution of all.

To check whether our results are affected by the aggregation of data from different years, we have also estimated α and σ for every single year during the period 1980-2011. As Figure 3 illustrates, there is a remarkable year-to-year stability of the Laplace distribution for profit rates, with rather small fluctuations in the parameter values over time. In 25 out of 32 years the estimated shape parameter is consistent with a Laplace distribution at the 95% confidence level. Since maximum likelihood estimates of the shape parameter are quite sensitive to outliers, we investigated the relatively small values of the shape parameter in the last four years, and it turns out that they are in fact due to very few extreme observations. Eliminating, for instance, merely the two most extreme profit rates at both sides of the spectrum leads to

⁸Some studies (see, for instance Amaral et al., 1997; Bottazzi et al., 2011) investigate the empirical distribution of “rescaled” growth rates that are divided by their standard deviations conditional on firm size. To a certain extent, this procedure brings the parameters estimates closer to a Laplace fit, yet we still obtain a significantly better Laplace fit for profit rates using merely the raw data on the ratio of operating income to total assets in our dataset.

⁹Similar estimates of the Subbotin shape parameter are reported by Bottazzi et al. (2011) for the distribution of sales growth rates of French manufacturing firms.

estimates for α that cannot be distinguished from the Laplace benchmark ($\alpha = 1$) at the 95% confidence level. As far as growth rates are concerned, we can reject the null hypothesis $\alpha = 1$ in approximately 87% (84%) of cases when firm size is measured in terms of sales (total assets), while in the case of employment growth rates the Laplacian null hypothesis is rejected for every single year. The distribution of market value growth rates is consistent with the Laplacian null hypothesis in 20 out of 31 years, however, the time evolution of the shape parameter is considerably more volatile than for any other variable, in particular profit rates. Moreover, the scale parameter of the growth rate distribution also exhibits pronounced fluctuations over time, suggesting that the Laplacian nature of the pooled market value growth rate distribution is an artefact of aggregation. We will continue under the assumption that the Laplace distribution is a reasonable benchmark for the distribution of profit rates, while growth rates are more leptokurtic.

2.3 Autocorrelations

Visual inspection of the line charts for a dozen randomly chosen time series of growth and profit rates in our sample indicates that profit rates are substantially more persistent than growth rates. To properly quantify this first impression, we consider here the autocorrelation function (acf)

$$\rho(\tau) = \frac{\gamma(\tau)}{\gamma(0)}, \quad (6)$$

where $\gamma(\cdot)$ denotes the autocovariance function, and τ is the time lag. Several estimators have been proposed for the autocovariance function. If the true mean is unknown, Hamilton (1994) suggests to use the estimator

$$\hat{\gamma}(\tau) = T^{-1} \sum_{t=1}^{T-\tau} (X_t - \bar{x}_T)(X_{t+\tau} - \bar{x}_T), \quad (7)$$

where

$$\bar{x}_T = T^{-1} \sum_{t=1}^T X_t \quad (8)$$

is the mean of a time series X_t with length T . For the estimation of the autocovariance function, however, we must consider that the number of observations per time series is quite small in our sample. In case of an autocorrelated process and a small number of observations, using the sample mean in equation (7) leads to a systematic underestimation of the true autocorrelation (see, for instance, Fuller, 1996). Intuitively, this negative bias stems from the fact that the autocorrelation coefficient is a scaled sum of cross-products of deviations of X_t from its mean. For each time series these deviations must sum to zero by construction, so that negative deviations must eventually be followed by positive deviations on average and vice versa. Therefore, the expected value of cross-products of deviations is negative (see Campbell et al., 1996). In order to mitigate this negative bias, we replace the estimated mean of each individual time series in (7) with the median $\hat{m}$ of the pooled

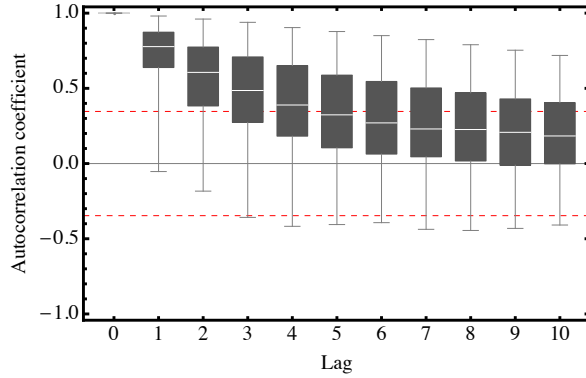


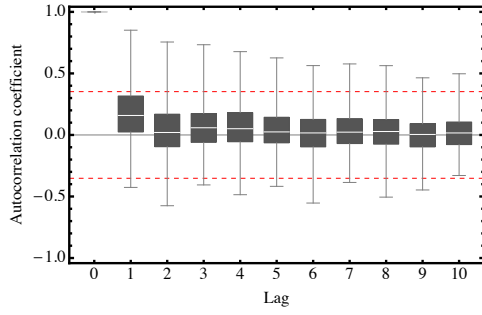
Figure 4: Box-and-whisker plot for the estimated autocorrelation functions of profit rates. The boxes include the 25 percent quantile, the median, and the 75 percent quantile. The red dashed lines show the 95 percent confidence interval under the null hypothesis of zero autocorrelations. The interval has been computed as $\pm 1.96/\sqrt{T}$, where $T = 32$ is the length of the profit rate time series.

empirical density, that is we always set $\bar{x}_T = \hat{m}$ in estimating the auto-correlation for each time series.¹⁰

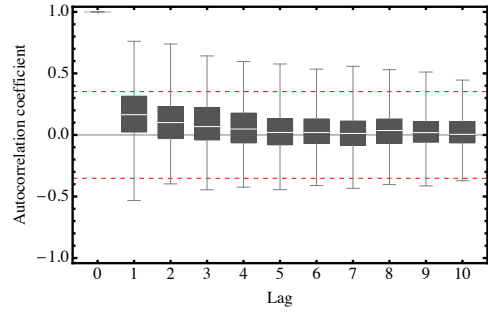
Figure 4 presents a box-and-whisker plot for the 522 estimated autocorrelation functions of the profit rate time series, while the corresponding results for the various growth rates are illustrated in Figure 5. Our analysis suggests that statistically significant autocorrelations in growth rates can only be found in relatively few time series: serial correlation seems to be completely absent in annual growth rates of market value (in line with the weak-form efficient market hypothesis of Fama, 1991), which is consistent with many previous findings in the pertinent empirical literature (see, for instance, Cont, 2001, for a review of the stylized facts of financial returns). The annual growth rates of sales, total assets, and employment appear to be slightly more persistent, yet the estimated autocorrelation coefficients cannot be distinguished from zero at the 95% confidence level in the vast majority of cases. If present at all, we find that autocorrelation in growth rates is very weak, consistent with previous results by Bottazzi et al. (2001) and Bottazzi and Secchi (2005). These findings can be interpreted as evidence against the “optimal size” hypothesis since one should observe pronounced positive autocorrelations in growth rates as firms approach some (system-wide) optimal size.

Profit rates, on the other hand, exhibit strong positive autocorrelations. Similar results have been reported in the so-called persistence of profits literature, which finds significantly positive first-order autoregressive coefficients in time-series regressions of profit rates (for a recent take on the subject see, for instance, Cable and Mueller, 2008). Notice, however, that these models typically approach the dynamics of firm profitability via stationary AR(1) processes, and hence are misspecified because their stationary distribution is Gaussian, yet the previous subsection shows that empirical profit rate distributions are much closer to the Laplace. Since we

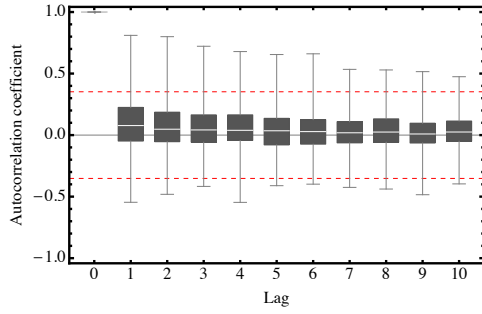
¹⁰The relevance of the negative bias for our subsequent diffusion model is illustrated in Figure 12 of appendix B. The reason why the particular substitution $\bar{x}_T = \hat{m}$ is preferable will be clarified in section 2.4, and is essentially based on the statistical equilibrium property of the profit rate time series.



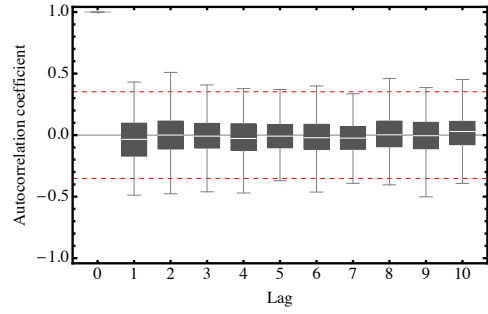
(a) Growth in sales



(b) Growth in total assets



(c) Growth in number of employees



(d) Growth in market value

Figure 5: Box-and-whisker plots for the estimated autocorrelation functions of firm growth rates. The boxes include the 25% quantile, the median, and the 75% quantile. The red dashed lines show the 95% confidence interval under the null hypothesis of zero autocorrelations. The interval has been computed as $\pm 1.96/\sqrt{T}$, where $T = 31$ is the length of the growth rate time series.

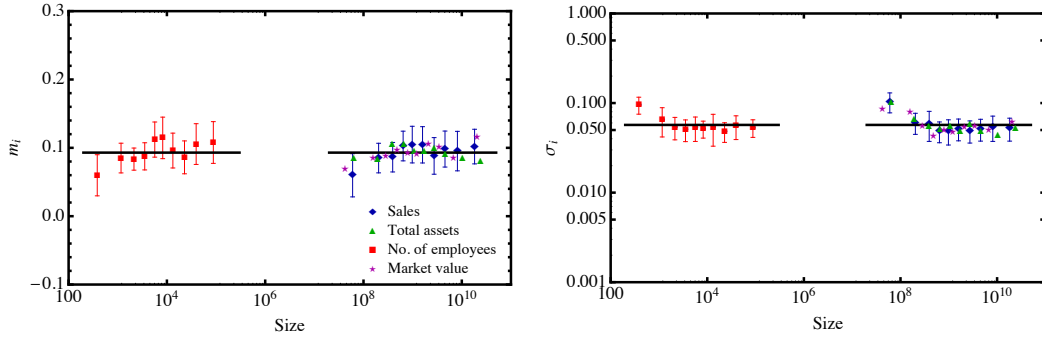


Figure 6: Location (m_i) and dispersion (σ_i) of firm profit rates as a function of firm size. Points represent binned data and have been computed in the following way: for each profit rate time series, $i = 1, \dots, 522$, we calculate a firm's median m_i and mean absolute deviation σ_i . Then we split the firms according to their median sizes into ten (almost) equipopulated bins. The points represent the average m_i and σ_i of the approximately 52 firms in each bin. Linear regressions indicate that a statistically significant relationship between size and average profitability (or size and profit dispersion) breaks down as soon as the first bin is excluded. Regression results are summarized in Tables 4 and 5 of appendix C. The horizontal lines represent the unconditional estimates of the location and dispersion parameters $\hat{m} = 0.093$ and $\hat{\sigma} = 0.057$ of the pooled empirical density of profit rates. Error bars corresponding to one median absolute deviation are shown for sales and number of employees (error bars for the other size definitions are nearly identical and have been omitted for visual clarity).

cannot rule out a negative bias in the estimated autocorrelation coefficients of profit rates, it seems imprudent to specify the number of statistically significant time lags for the non-parametric analysis in Figure 4. Instead we will introduce the correlation time of our subsequent diffusion model as an alternative measure of profit persistence in section 4.1.

2.4 Size (in)dependence of m and σ

The law of proportionate effect is conventionally understood as a multiplicative stochastic process whereby a firm's current size is the result of a sequence of independent growth shocks. According to the central limit theorem, the growth rate distribution should then be Gaussian, and the corresponding firm size distribution should be log-normal. While the hypothesis of proportionate random growth is useful to explain the considerable heterogeneity in firm size, it still lacks an economic justification, or as Sutton (1997, p. 42) puts it, “[t]here is no obvious rationale for positing any general relationship between a firm's size and its expected growth rate.”

There is, on the other hand, good reason for profit rates to be independent of size.¹¹ Profit rates are central to economic competition since they guide the allocation of capital across competing uses in different sectors and industries. Capital seeks out abnormally profitable activities independent of their size, because it is the rate of return to invested capital (say, ten percent), and not the absolute return (say, ten million currency units) that guides the allocation of capital. In the absence of further information, one should therefore expect both the location parameter m

¹¹That does not mean, however, that profit rates will be independent of each other. After all, the distribution of profit rates is not Gaussian and therefore strongly suggests that the independence assumption of the central limit theorem is violated.

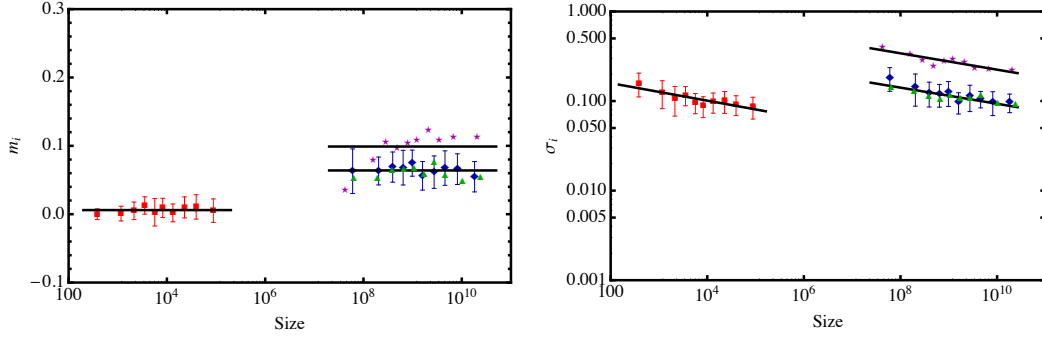


Figure 7: Location (m_i) and dispersion (σ_i) of growth rates as a function of firm size. The binning procedure is the same as in Figure 6. Least squares power law fits for the relation between the dispersion of growth rates and size yield the following scaling exponents: -0.10 ± 0.02 for sales, -0.06 ± 0.01 for total assets, -0.10 ± 0.02 for the number of employees, and -0.09 ± 0.02 for market value. The black solid lines in the right panel have slopes of -0.1 (growth in number of employees) and -0.08 (growth in sales, total assets, and market value) respectively. Error bars corresponding to one median absolute deviation are shown for sales and number of employees, while values for the other size definitions are nearly identical and have been omitted for visual clarity.

of the profit rate distribution, and the dispersion parameter σ to be independent of firm size. In order to judge how well the data reflect this prediction, we consider the median and mean absolute deviation as the location and dispersion measures, because they correspond to the maximum likelihood estimators of m and σ when sampling from a Laplace distribution (see, for instance, Johnson et al., 1995; Kotz et al., 2001). As illustrated in Figure 3, the Laplace is a reasonable benchmark for the pooled profit rate distribution, and hence we denote the parameter estimates from the pooled cross-sectional distribution by $\hat{m}$ and $\hat{\sigma}$.

To further fix notation, let m_i and σ_i denote the median and mean absolute deviation of the profit rate time series of firm i . Figure 6 suggests that both the median and mean absolute deviation of profit rates are rather homogeneous across different size classes, and are reasonably close to the unconditional values $\hat{m} = 0.093$ and $\hat{\sigma} = 0.057$ of the pooled empirical profit rate distribution. We cannot rule out the existence of a small negative bias for the smallest size bin, yet this bias is caused by around 15 to 25 out of 522 firms, that is by about 3 to 5 percent of all long-lived companies.¹² The visual impression that m_i and σ_i are virtually the same for the vast majority of firms is mostly confirmed by linear regressions, which yield slope coefficients that cannot be distinguished from zero at the usual confidence levels once the smallest size bin is excluded from the analysis (see Tables 4 and 5 in the appendix for details). The intercept in the linear regressions provides rather limited information since firm sizes span several orders of magnitude and are very large to begin with, so that an extrapolation of any linear relationship to size zero is hardly meaningful. In any case, the values of m_i and σ_i in each bin cannot be distinguished from $\hat{m}$ and $\hat{\sigma}$ at the usual significance levels.¹³ The remarkable similarity between

¹²Twelve (thirteen) of the twenty-five corporations with the lowest m_i (highest σ_i) operate in just four industries with SIC codes 13, 36, 38, and 67.

¹³Notice that one median absolute deviation error bars, as shown in Figure 6, correspond to a 63% confidence interval when sampling from a Laplace distribution, while $\pm 2\sigma \approx 87\%$ and $\pm 3\sigma \approx 95\%$.

the parameters of the cross-sectional distribution and the individual location and dispersion parameters of firm-level time series suggests that profit rates are ergodic and stationary.

Figure 7 repeats the analysis for the growth rates of firm size. While the location parameter of the growth rates is not markedly affected by size, we find a clear inverse relation between the dispersion of growth rates and company size, in line with previous studies that report power-laws with scaling exponents close to -0.15 (see, for instance, Amaral et al., 1997; Bottazzi and Secchi, 2003; Stanley et al., 1996). Fitting a power law to our data yields scaling exponents ranging from -0.06 ± 0.01 for growth in total assets to -0.10 ± 0.02 for sales and employment growth rates. In contrast to profit rates, removing the smallest size bin from the growth rate analysis does not lead to significantly different slope coefficients,¹⁴ and preserves the scaling of the dispersion of growth rates with firm size, which the pertinent literature typically ascribes to firm diversification.

3 Model

The preceding analysis suggests that profit rates are characterized by a stationary cross-sectional distribution, and that firm-level time series exhibit persistent autocorrelations. In addition, the location and dispersion of the individual series are independent of size and very close to the location and dispersion estimates $\hat{m}$ and $\hat{\sigma}$ of the cross-sectional distribution. These properties of corporate profit rates establish a major difference to growth rates, and would seem to represent a more immediate way to study the competitive behavior of corporations, at the very least from a statistical point of view. Inspired by the empirical densities of cross-sectional profit rates, Alfarano et al. (2012) have recently introduced a diffusion process with a stationary Laplace distribution. We argue here that their process is not only consistent with the observed cross-sectional distribution, but also with the time series properties of surviving corporations, including their autocorrelation structures.

3.1 Diffusion

They propose the stochastic differential equation

$$dX_t = -\frac{D}{2\sigma} \text{sign}(X_t - m) dt + \sqrt{D} dW_t, \quad (9)$$

to model the dynamic evolution of firm profitability, where X_t denotes the profit rate, σ is a dispersion parameter, $\text{sign}(\cdot)$ denotes the signum function, m is the average rate of profit, and dW_t are Wiener increments. The (square root of the) constant term D determines the noise level in the (random) second term, but notice that it also influences the strength of the reversion to m in the (deterministic) first term. From an economic point of view, this part of the stochastic process reflects the negative feedback mechanism of classical competition: capital will seek out sectors or industries where the profit rate is higher than the economy-wide average, typically attracting labor, raising output, and reducing prices and profit rates in the sector. This provides an incentive for capital to leave the sector, leading in turn to higher

¹⁴To be precise, the cited papers consider the relationship between size and the standard deviation of growth rates. We have also estimated the power law exponents for this relationship and find that the results are very similar to the ones we report for the mean absolute deviation.

prices and profit rates for the surviving firms. Notice that the first term (or *drift function*) of the stochastic model (9) does not depend on the current profitability of a firm. The second term (or *diffusion function*) is governed by random Wiener increments, which incorporate all idiosyncratic factors affecting firm profitability. In the particular case (9), the noise level $\sqrt{D}$ is constant over time and independent of the profit rate. Another particular feature of the diffusion (9) is that the drift and the diffusion function are intertwined, since the variance of the idiosyncratic noise term affects the speed of adjustment towards the system-wide average. It can be interpreted in the sense that competition simultaneously generates fluctuations in individual corporate destinies and convergence to the average rate of profit.

The notion of statistical equilibrium rests on the idea that all surviving corporations are subject to the same stochastic process (9), with common parameters m and σ that match the location and dispersion parameters of the stationary distribution of the process,

$$f_S(x; m, \sigma) = \frac{1}{2\sigma} \exp \left(- \left| \frac{x - m}{\sigma} \right| \right). \quad (10)$$

It is easily verified that (10) is obtained from (5) for $\alpha = 1$. The empirical analysis in the preceding section therefore suggests that m and σ can be readily observed from the pooled empirical density of profit rates, and would apply to the individual destinies of all surviving corporations, regardless of their size or industry. In other words, statistical equilibrium describes a situation where the profit rate of each surviving corporation reverts to the same *systemic* rate of profit, and fluctuates around it with the same (systemic) variability. That means that the only source of firm-specific effects in the model originates from the diffusion coefficient D , because the unconditional equilibrium distribution (10) does not depend on this parameter. Accordingly, the statistical equilibrium model leaves a single degree of freedom for idiosyncrasies in corporate profitability.

3.2 Transient density

A useful alternative representation of a diffusion process is provided by its transient density (or Fokker-Planck equation, see Risken, 1996), which describes the time evolution of the stochastic system by means of a second-order partial differential equation,

$$\frac{\partial p(x, t)}{\partial t} = - \frac{\partial}{\partial x} (A(x; D) p(x, t)) + \frac{1}{2} \frac{\partial^2}{\partial x^2} (B(x; D) p(x, t)), \quad (11)$$

where $A(x; D)$ and $B(x; D)$ are the drift and diffusion functions of the underlying diffusion process, and

$$p(x, t) = f(x, t | x_0, t_0) \quad (12)$$

denotes the conditional probability density for a transition from state x_0 at time $t_0 = 0$ to state x at time t . For the particular diffusion (9) with zero mean, that is

$$Z_t = X_t - m, \quad (13)$$

and initial condition $f(z, 0|z_0, 0) = \delta(z - z_0)$, where $\delta(\cdot)$ denotes Dirac's delta function, Toda (2012) demonstrates that a closed-form solution to (11) exists and is given by

$$f(z, t|z_0, 0) = \frac{1}{\sqrt{2D\pi t}} \cdot \exp\left(-\frac{(z - z_0)^2}{2Dt} - \frac{1}{2\sigma}(|z| - |z_0|) - \frac{D}{8\sigma^2}t\right) + \frac{1}{2\sigma} \exp\left(-\frac{1}{\sigma}|z|\right) \Phi\left(-\frac{|z| + |z_0| - (Dt)/(2\sigma)}{\sqrt{Dt}}\right), \quad (14)$$

where $\Phi(\cdot)$ denotes the cumulative distribution function of the standard normal. The closed-form solution of the Fokker-Planck equation in (14) allows us to estimate the idiosyncratic diffusion coefficient by maximum likelihood, and it is also helpful in finding a closed-form solution for the autocorrelation function of the diffusion process (9).

3.3 Autocorrelation function

For stationary Markov processes, the autocorrelation function obeys the textbook formula (see, for instance, van Kampen, 1992)

$$\kappa(\tau) = \int_{-\infty}^{\infty} dz \int_{-\infty}^{\infty} dz_0 \, z z_0 f(z, \tau|z_0, 0) f_S(z_0), \quad (15)$$

where f_S denotes the stationary density. Here the stationary density corresponds to (the zero-mean-shifted version of) equation (10), and the transient density f obeys (14). In this case, Touchette et al. (2010) show that the autocorrelation function of (9) is characterized by an (asymptotic) exponential decay,¹⁵

$$\kappa(\tau) = \frac{1}{6\sqrt{\frac{2\pi D\tau}{\sigma^2}}} \exp\left(-\frac{D\tau}{8\sigma^2}\right) \left\{ \left(\frac{\sqrt{\frac{\pi D\tau}{2\sigma^2}}}{2} \exp\left(\frac{D\tau}{8\sigma^2}\right) \operatorname{erfc}\left(\frac{\sqrt{\frac{D\tau}{\sigma^2}}}{2\sqrt{2}}\right) - 1 \right) \left(\frac{D^3\tau^3}{8\sigma^6} + \frac{3D^2\tau^2}{2\sigma^4} - \frac{6D\tau}{\sigma^2} + 24 \right) + \frac{D^2\tau^2}{2\sigma^4} + 24 \right\}. \quad (16)$$

Figure 11 in the appendix illustrates the probability density and autocorrelation function of simulated realizations of the diffusion process in equation (9). While the model is consistent with the distributional and autocorrelation properties of empirical profit rates, the good fit between the estimated and theoretical autocorrelation function only occurs for time series that are sufficiently long. For shorter time series, we do observe a negative bias in the estimated autocorrelation function. This, however, implies that the persistence of abnormal profits is actually even stronger than Figure 4 suggests. If (9) provides a meaningful description of firm profitability, we can avoid the negative bias in the persistence of abnormal profits by using the theoretical autocorrelation function. Estimating the persistence of abnormal profits then boils down to estimating the diffusion coefficient D from the transient density (14).

¹⁵The pre-factor stems from the non-linear nature of the drift function in (9).

4 Results

In the diffusion model (9), the persistence of profits is determined by the drift function, hence the speed of convergence towards the systemic rate of profit depends on two parameters: the diffusion coefficient D and the scale parameter σ . In statistical equilibrium all firms are subject to the same location and scale parameters m and σ , so the diffusion coefficient D remains as the only source of idiosyncratic differences in the profitability of surviving corporations. If σ is the same for all corporations, then the noise level D_i measures the persistence of abnormal profits directly, and can be interpreted in the sense that firms with larger diffusion coefficients are prone to larger shocks in their profitability, while their abnormal profits do not persist for long. Conversely, firms with smaller diffusion coefficients are on average subject to smaller shocks, while their abnormal profits are more persistent. In order to estimate the diffusion coefficient for each profit rate series, we apply the maximum likelihood method to the solution (14) of the Fokker-Planck equation.

4.1 Estimation of the diffusion coefficient

Given discrete annual observations, we estimate the diffusion coefficients for each firm by numerically minimizing the negative log-likelihood

$$-\log \mathcal{L}(D_i) = -\log f_S(z_{i,0}) - \sum_{t=0}^{T-1} \log f(z_{i,t+1}|z_{i,t}; D_i) \quad (17)$$

with respect to D_i , where $f_S(z_{i,0})$ is the stationary Laplace density of some initial state $z_{i,0}$, and $f(z_{i,t+1}|z_{i,t}; D_i)$ is the solution of the transient density (14) evaluated for each observation $z_{i,t+1} = p_{i,t+1} - \hat{m}$ at time $t + 1$ conditional on the previous observation $z_{i,t}$ at time t . Equipped with the estimated coefficients, we then compute the *speed of adjustment* (or *characteristic time scale* or *relaxation time*) of the profit diffusion from (16) as the number of years that are necessary for the autocorrelation function to reach the value one half.¹⁶

Figure 8 presents the estimated diffusion coefficients and corresponding half-life of abnormal profits, where we observe a pronounced variability in the diffusion coefficients that translates into heterogeneous time horizons for the dissipation of abnormal profits. The median diffusion coefficient is $D_{med} \approx 1.1 \times 10^{-3}$, implying a standard deviation of the idiosyncratic noise in the diffusion equation of $\sqrt{D_{med}} \approx 3.3\%$ per annum, which corresponds to a longitudinal relaxation time of about nine years. For some firms, however, the diffusion coefficients are very small and imply relaxation times that are much longer than the length of the observed time series. Analyzing these firms in more detail, we find that considerable fractions are made up by utilities or insurance companies, with total assets far above the sample average, high capital intensity, and relatively steady profit series in comparison to other sectors. At this point we can merely speculate that entry and exit barriers in these sectors, probably stemming from large capital requirements, prevent a smooth and frictionless reallocation of capital in search of profit rate equalization.

Firms with relatively short relaxation times, on the other hand, disproportionately often operate in business sectors with SIC code 36 (electronical equipment),

¹⁶We consider the half-life definition in order to account for the non-linear nature of the diffusion process. The usual choices that are typically based on the dominant exponential term would in fact neglect such effects.

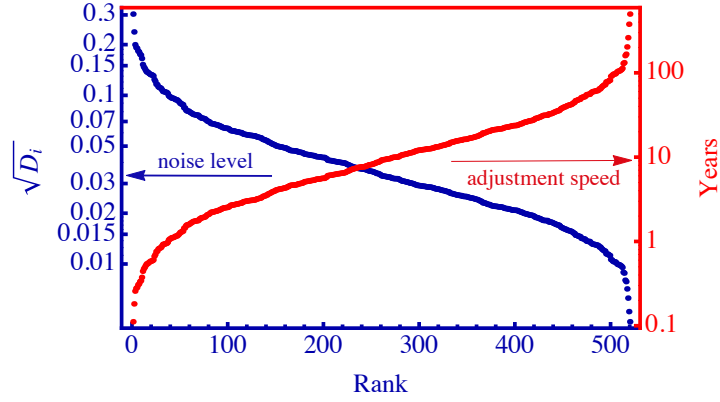


Figure 8: Sorted estimates of the diffusion coefficient (left axis) and corresponding relaxation times of abnormal profits (right axis) for long-lived US corporations. The latter shows the number of years that are necessary for the autocorrelation function (15) to reach the value 1/2. The arrows indicate the median noise level of around 3.3% p.a., and the corresponding median half-life of abnormal profits (around 9 years).

38 (measuring instruments), and 13 (oil and gas extraction). Intuitively, the latter is characterized by a high degree of uncertainty, while large changes in profitability for the former two sectors might be caused by operating leverage effects.¹⁷

4.2 What determines the diffusion coefficient?

The observed heterogeneity in the diffusion coefficient raises the question whether firm or industry characteristics affect the persistence of abnormal profits. We will focus on what are perhaps merely the most obvious attributes, and consider here the impact of size, diversification, and capital intensity on the persistence of abnormal profits.

4.2.1 Firm size

While the data suggest that size basically does not influence the rate of profit, we can ask whether size instead has an impact on the diffusion coefficient? On average larger corporations appear more stable and are affected by smaller idiosyncratic shocks to their profitability than smaller entities. The double-logarithmic plot in Figure 9 suggests that the standard deviation of the idiosyncratic noise scales with size according to a power law

$$\sqrt{D} \sim \alpha S^{-\beta}. \quad (18)$$

¹⁷Operating leverage increases with the proportion of fixed in relation to variable operating costs. During demand surges, high operating leverage could well lead to larger profits, but it also makes firms more vulnerable as they cannot readily cut expenses to absorb plummeting demand when most costs are tied up in machinery, plants, real estate, or distribution networks.



Figure 9: Standard deviation of the idiosyncratic noise term in the Laplace diffusion vs firm size on double-logarithmic scale. Error bars corresponding to one median absolute deviation are shown for sales and number of employees, while the values for the other quantities are very similar and have been omitted for visual clarity. Least-squares power-law fits yield a scaling exponent of 0.17 ± 0.03 for sales, 0.18 ± 0.03 for total assets, 0.16 ± 0.02 for the number of employees, and 0.16 ± 0.03 for market value. The black solid lines are regression fits with slopes of -0.16 (left) and -0.17 (right).

To avoid distortions arising from booms and busts in single years, we calculated the mean size of each surviving corporation during the period 1980-2011,¹⁸ and divided the sorted values into deciles, calculating the median size and $\sqrt{D_i}$ in each decile. Fitting a power relation

$$\log \sqrt{D} = \log \alpha - \beta \cdot s \quad (19)$$

to the data yields least squares estimates of $\beta = 0.17 \pm 0.03$ for sales, $\beta = 0.18 \pm 0.03$ for total assets, $\beta = 0.16 \pm 0.02$ for the number of employees, and $\beta = 0.16 \pm 0.03$ for market value. All estimates are significant at the one percent level and indicate an inverse relationship between size and the noise level, so the larger a corporation the more persistent its abnormal profits tend to be.

4.2.2 Diversification

In order to proxy the degree of corporate diversification, we consider Datastream's product segment decomposition of corporate revenues. The data associate segment-level SIC codes with the corresponding revenues of each company, and we use the product segment data to compute three common measures of corporate diversification: segment count, Herfindahl index, and entropy.

The first measure literally counts the number of sectors a company operates in. Since Datastream merely provides up to ten business segments per company, we decided to group business sectors on a 3-digit SIC level. Table 3 illustrates that 66 of the 522 corporations concentrate their business activity in a single sector, while the remaining 456 companies are diversified across different sectors. Around half of the sample operates in four business segments or more. Considering the median D in the third column of Table 3, we observe a tendency for the diffusion coefficient to

¹⁸Taking either the latest available observation of firm size or the maximum value per firm leads to very similar results.

decrease with the number of business segments. To further quantify this impression, we have tested for differences between medians in the different groups. Comparing firms operating in one business segment with companies that are active in four (or more) sectors, a Mann-Whitney test rejects the null hypothesis that the average diffusion coefficient of non-diversified corporations is smaller or equal to that of diversified ones at the five percent level.

The business segment count, however, lacks information on the relative importance of the different segments, that is on how much the respective revenues in these segments contribute to a corporation's overall sales. Therefore, Herfindahl suggests a diversification index that computes the sum of squared shares of each segment's contribution to total sales

$$H_i = \sum_{j=1}^n P_{ij}^2, \quad (20)$$

where P_{ij} is the percentage share of company i 's sales that is generated in business segment j . Notice that the measure decreases with increasing diversification. Alternatively, the entropy methodology can be applied to calculate a sales diversity index:¹⁹

$$E_i = - \sum_{j=1}^n P_{ij} \log P_{ij}. \quad (21)$$

Unlike the Herfindahl index, which weighs the share of each business segment by itself, the entropy measure weighs each P_j by the logarithm of $1/P_j$, so that it is more sensitive to small sales shares than the Herfindahl index, and largely ignores small differences in large sectors. The entropy measure in equation (21) increases with increasing diversification.

The Spearman rank correlation coefficient for the relationship between $\sqrt{D_i}$ and the Herfindahl-index is 0.17, indicating a moderate negative effect of diversification on the adjustment speed of the process.²⁰ Based on the Spearman rank test,

Table 3: Median diffusion coefficient (and its square root) for firms that operate in a certain number of business segments (3-digit SIC level). The persistence of abnormal profits seems to increase with the business segment count of a corporation.

Business segment count	Number of firms	D_{med}	$\sqrt{D_{med}}$
1	66	0.001764	0.042
2	99	0.001681	0.041
3	97	0.001369	0.037
4	101	0.000841	0.029
5	70	0.000961	0.031
6	49	0.000784	0.028
7	29	0.001024	0.032
8	11	0.000625	0.025

¹⁹For instance, Horowitz (1970) uses entropy as a measure of industry concentration, while Jacquemin and Berry (1979) use the concept to measure corporate diversification.

²⁰Again, notice that more diversification leads to a reduction of the Herfindahl index, thus a positive correlation between D_i and the Herfindahl index implies that the speed of convergence toward the average profit rate decreases with increasing diversification.

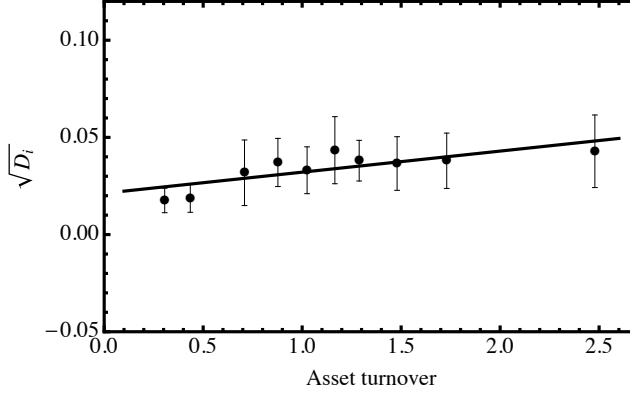


Figure 10: Standard deviation of the idiosyncratic noise as a function of asset turnover, measured as the ratio of sales to total assets. Ordinary least squares regression of $\sqrt{D_i}$ on asset turnover yields an intercept of 0.0227 ± 0.004 and a slope parameter of 0.0112 ± 0.003 . Both parameters are statistically significant at the one percent level; error bars represent one median absolute deviation.

we find that the null hypothesis of $\sqrt{D_i}$ and the Herfindahl index being independent or negatively correlated can be rejected at the one percent level.

In case of the entropy measure, the correlation coefficient equals -0.15 and the null hypothesis that $\sqrt{D_i}$ and the entropy measure are not negatively correlated is also rejected at the 1 percent level. Furthermore, we have regressed the square root of the diffusion coefficient on these two diversification measures and find coefficients of 0.0246 ± 0.006 for the Herfindahl-index and -0.0142 ± 0.0038 for the entropy measure. Both coefficients are statistically significant at the one percent level.

Overall, the results suggest that there is a moderate negative correlation between the diffusion coefficient and firm diversification, which is nevertheless quite robust with respect to several diversification measures. Abnormal profits would thus seem to be more persistent for more diversified corporations.

4.2.3 Intensity of capital

The long relaxation times for insurance and utilities corporations indicate that capital intensity has an impact on the persistence of abnormal profits. According to the DuPont identity, the profit rate of a firm i can be decomposed into the product of its profit margin and asset turnover,

$$p_{i,t} = \frac{I_{i,t}}{S_{i,t}} \cdot \frac{S_{i,t}}{A_{i,t}}, \quad (22)$$

where (the inverse of) the latter measures capital intensity. The coefficient for the rank correlation between asset turnover and the noise level $\sqrt{D_i}$ equals 0.27 , implying that abnormal profits are more persistent for capital intensive corporations (with a correspondingly low asset turnover). A one-sided Spearman rank test rejects the null hypothesis of nonpositive correlations at the one percent level.

Figure 10 provides an alternative illustration of the relationship between asset turnover and the noise level. We calculate the average asset turnover for each corporation during the period 1980-2011, group them into deciles, and calculate the

median for each bin as well as the median of the associated noise levels $\sqrt{D_i}$. An ordinary least squares regression yields an intercept of 0.0227 ± 0.004 and a slope parameter of 0.0112 ± 0.003 that are both statistically significant at the one percent level. In summary the abnormal profits of capital intensive corporations would appear to be more persistent.

5 Discussion

The possibly most fundamental questions, pertaining to the origin of the particular values of the systemic rate of profit and its dispersion, still remain unanswered here. Yet the data suggest that statistical equilibrium provides a reasonable first approximation to the profitability of surviving corporations. Conditional on survival, US corporations generate an average rate of profit of about nine percent, along with a rather tranquil playground that disperses profit rates by less than six percent on average. Therefore it would appear that survival by itself warrants some sort of autopilot mode for corporations, in which they cannot do better but, perhaps surprisingly, also not worse than the system-wide average. Consequently, the idiosyncratic characteristics of corporations are independent of the systemic rate of profit and merely have an impact on how quickly abnormal profits are dissipated.

There are undeniably second-order effects that are not accounted for by the diffusion model. A substantial fraction of the deviations reported in sections 2 and 4 can be traced to a relatively small number of corporations in even fewer industries: about a dozen corporations with SIC codes 13 (oil and gas extraction), 36 (electronic equipment excluding computers), 38 (measuring instruments), and 67 (holding and other investment offices) account for half the deviations, which stem from the high volatility and leptokurtosis in the respective corporate time series. At the other end of the spectrum the largest corporations, like insurance carriers and utility companies, exhibit the least volatility and kurtosis in their profit rate series, and therefore account for most of the deviations in the relaxation time of abnormal profits. Recall that their estimated adjustment speeds exceed the length of the observed series by almost an order of magnitude. So Baumol's idea that the most capitalized corporations are somehow privileged in a competitive environment lives on in a modified form, however not relating to the rate of profit itself, but rather to the long persistence of abnormal profits in capital intensive industries.²¹

Pronounced deviations from the diffusion model might help to identify imperfections in the competitive environment, and potentially have antitrust implications. Since the diffusion model rests on the classical idea of a perpetual reallocation of capital in search of profit rate equalization, large empirical deviations in profit persistence should essentially be tied to frictions in the reallocation of capital.

Finally, and maybe most controversially, we would like to argue that concerns of a 'survivorship bias' are perhaps the wrong way of framing the empirical analysis. After all, Gabaix's granular view of aggregate fluctuations in the US economy firmly suggests that the surviving corporations in our sample account for the major share of macroeconomic fluctuations, and are thus at the very least an interesting group of firms to study in its own right. Since everything in our analysis is conditional on (the uncertain and unpredictable) survival of corporations, one might instead

²¹Notice that stakeholders in these industries, once their profitability actually happens to be *below* the systemic rate, might not consider it such a privilege after all.

wonder whether there is a systemic cost for the survival of a certain number of large corporations? Does capital need to be churned, do other corporations have to die, in order to observe the tranquil dissipation of excess profits for a certain (and ultimately interchangeable) set of surviving corporations? And if so, how much capital needs to be metabolized in the process?

References

- Alexander, S. S. (1949). The effect of size of manufacturing corporation on the distribution of the rate of return. *The Review of Economics and Statistics*, 31(3):229–235.
- Alfarano, S., Milaković, M., Irle, A., and Kauschke, J. (2012). A statistical equilibrium model of competitive firms. *Journal of Economic Dynamics and Control*, 36(1):136–149.
- Almus, M. and Nerlinger, E. A. (2000). Testing Gibrat’s Law for young firms - empirical results for West Germany. *Small Business Economics*, 15(1):1–12.
- Amaral, L. A. N., Buldyrev, S. V., Havlin, S., Maass, P., Salinger, M. A., Stanley, H. E., and Stanley, M. H. R. (1997). Scaling behavior in economics: The problem of quantifying company growth. *Physica A*, 244(1):1–24.
- Amato, L. and Wilder, R. P. (1985). The effects of firm size on profit rates in U.S. manufacturing. *Southern Economic Journal*, 52(1):181–190.
- Axtell, R. L. (2001). Zipf distribution of U.S. firm sizes. *Science*, 293(5536):1818–1820.
- Baumol, W. J. (1967). *Business behavior, value and growth*. Harcourt, Brace & World, New York, NY.
- Boeri, T. and Cramer, U. (1992). Employment growth, incumbents and entrants. *International Journal of Industrial Organization*, 10(4):545–565.
- Bottazzi, G., Cefis, E., and Dosi, G. (2002). Corporate growth and industrial structures: Some evidence from the Italian manufacturing industry. *Industrial and Corporate Change*, 11(4):705–723.
- Bottazzi, G., Coad, A., Jacoby, N., and Secchi, A. (2011). Corporate growth and industrial dynamics: Evidence from French manufacturing. *Applied Economics*, 43(1):103–116.
- Bottazzi, G., Dosi, G., Lippi, M., Pammolli, F., and Riccaboni, M. (2001). Innovation and corporate growth in the evolution of the drug industry. *International Journal of Industrial Organization*, 19(7):1161–1187.
- Bottazzi, G. and Secchi, A. (2003). Common properties and sectoral specificities in the dynamics of U.S. manufacturing companies. *Review of Industrial Organization*, 23(3):217–232.

- Bottazzi, G. and Secchi, A. (2005). Growth and diversification patterns of the worldwide pharmaceutical industry. *Review of Industrial Organization*, 26(2):195–216.
- Bottazzi, G. and Secchi, A. (2006). Explaining the distribution of firm growth rates. *The RAND Journal of Economics*, 37(2):235–256.
- Cable, J. R. and Mueller, D. C. (2008). Testing for persistence of profits’ differences across firms. *International Journal of the Economics of Business*, 15(2):201–228.
- Campbell, J. Y., Lo, A. W., and MacKinlay, A. C. (1996). *The econometrics of financial markets*. Princeton University Press, Princeton, NJ.
- Chesher, A. (1979). Testing the Law of Proportionate Effect. *The Journal of Industrial Economics*, 27(4):403–411.
- Coad, A. (2007). A closer look at serial growth rate correlation. *Review of Industrial Organization*, 31(1):69–82.
- Cont, R. (2001). Empirical properties of asset returns: Stylized facts and statistical issues. *Quantitative Finance*, 1(2):223–236.
- Fama, E. F. (1991). Efficient capital markets: II. *The Journal of Finance*, 46(5):1575–1617.
- Foley, D. K. (1994). A statistical equilibrium theory of markets. *Journal of Economic Theory*, 62(2):321–345.
- Foley, D. K. (2006). *Adam’s Fallacy*. Harvard University Press, Cambridge, MA.
- Fuller, W. A. (1996). *Introduction to statistical time series*. Wiley, New York, NY.
- Gabaix, X. (2011). The granular origins of aggregate fluctuations. *Econometrica*, 79(3):733–772.
- Garibaldi, U. and Scalas, E. (2010). *Finitary probabilistic methods in econophysics*. Cambridge University Press, Cambridge, UK.
- Geroski, P. A., Machin, S. J., and Walters, C. F. (1997). Corporate growth and profitability. *The Journal of Industrial Economics*, 45(2):171–189.
- Geroski, P. A. and Mazzucato, M. (2002). Learning and the sources of corporate growth. *Industrial and Corporate Change*, 11(4):623–644.
- Gibrat, R. (1931). *Les inégalités économiques*. Recueil Sirey, Paris.
- Goddard, J., Tavakoli, M., and Wilson, J. O. S. (2005). Determinants of profitability in European manufacturing and services: Evidence from a dynamic panel model. *Applied Financial Economics*, 15(18):1269–1282.
- Goddard, J., Wilson, J., and Blandon, P. (2002). Panel tests of Gibrat’s Law for Japanese manufacturing. *International Journal of Industrial Organization*, 20(3):415–433.
- Hall, M. and Weiss, L. (1967). Firm size and profitability. *The Review of Economics and Statistics*, 49(3):319–331.

- Hamilton, J. D. (1994). *Time Series Analysis*. Princeton University Press, Princeton, NJ.
- Hart, P. E. and Oulton, N. (1996). Growth and size of firms. *Economic Journal*, 106(438):1242–52.
- Horowitz, I. (1970). Employment concentration in the common market: An entropy approach. *Journal of the Royal Statistical Society*, 133(3):463–479.
- Ijiri, Y. and Simon, H. A. (1977). *Skew distributions and the sizes of business firms*. North-Holland, Amsterdam.
- Jacquemin, A. P. and Berry, C. H. (1979). Entropy measure of diversification and corporate growth. *The Journal of Industrial Economics*, 27(4):359–369.
- Johnson, N. L., Kotz, S., and Balakrishnan, N. (1995). *Continuous univariate distribution: 2*. Wiley, New York, NY.
- Kotz, S., Kozubowski, T., and Podgórski, K. (2001). *The Laplace distribution and generalizations: A revisit with applications to communications, economics, engineering, and finance*. Birkhäuser, New York, NY.
- Lotti, F., Santarelli, E., and Vivarelli, M. (2003). Does Gibrat’s Law hold among young, small firms? *Journal of Evolutionary Economics*, 13(3):213–235.
- Marcus, M. (1969). Profitability and size of firm: Some further evidence. *The Review of Economics and Statistics*, 51(1):104–107.
- Mundt, P., Förster, N., Alfarano, S., and Milaković, M. (2013). The real versus the financial economy: A global tale of stability versus volatility. Economics Discussion Papers 2013-8, Kiel Institute for the World Economy.
- Risken, H. (1996). *The Fokker-Planck equation: Methods of solutions and applications*. Springer, Berlin.
- Santarelli, E., Klomp, L., and Thurik, A. R. (2006). Gibrat’s law: An overview of the empirical literature. In Santarelli, E., editor, *Entrepreneurship, Growth, and Innovation*, volume 12 of *International Studies in Entrepreneurship*, pages 41–73. Springer, New York, NY.
- Simon, H. A. and Bonini, C. P. (1958). The size distribution of business firms. *The American Economic Review*, 48(4):607–617.
- Stanley, M. H. R., Amaral, L. A. N., Buldyrev, S. V., Havlin, S., Leschhorn, H., Maass, P., Salinger, M. A., and Stanley, H. E. (1996). Scaling behaviour in the growth of companies. *Nature*, 379(29):804–806.
- Stanley, M. H. R., Buldyrev, S. V., Havlin, S., Mantegna, R. N., Salinger, M. A., and Stanley, H. E. (1995). Zipf plots and the size distribution of firms. *Economics Letters*, 49(4):453–457.
- Subbotin, M. T. (1923). On the law of frequency of errors. *Matematicheskii Sbornik*, 31:296–301.
- Sutton, J. (1997). Gibrat’s legacy. *Journal of Economic Literature*, 35(1):40–59.

- Toda, A. A. (2012). The double power law in income distribution: Explanations and evidence. *Journal of Economic Behavior and Organization*, 84(1):364–381.
- Touchette, H., van der Straeten, E., and Just, W. (2010). Brownian motion with dry friction: Fokker-Planck approach. *Journal of Physics A: Mathematical and Theoretical*, 43(44):1–21.
- van Kampen, N. G. (1992). *Stochastic processes in physics and chemistry*. North-Holland, Amsterdam.
- Weiss, C. R. (1998). Size, growth, and survival in the upper Austrian farm sector. *Small Business Economics*, 10(4):305–312.
- Whittington, G. (1980). The profitability and size of United Kingdom companies, 1960-74. *The Journal of Industrial Economics*, 28(4):335–352.

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B Simulations

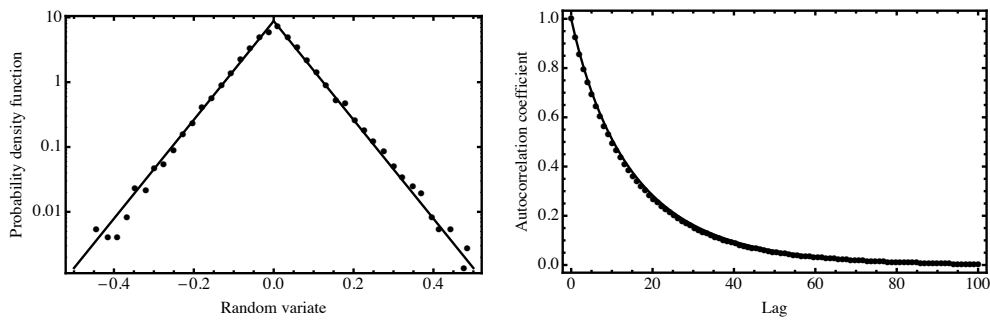


Figure 11: Simulation of the model by Alfaro et al. (2012). The left panel illustrates the probability density (black dots) of 1000 simulated time series with common parameters $\sigma = 0.057$, $D = 0.001$, and zero mean. The empirical distribution of the simulated data fits the probability density function of the Laplace distribution with identical location and scale parameters, which is represented by the black solid line. The right panel shows the estimated autocorrelation function of the diffusion process (black dots) together with the theoretical autocorrelation function, represented by the black solid line. The estimated autocorrelation function has been averaged over 1000 realizations of the diffusion process with time series length $T = 10000$.

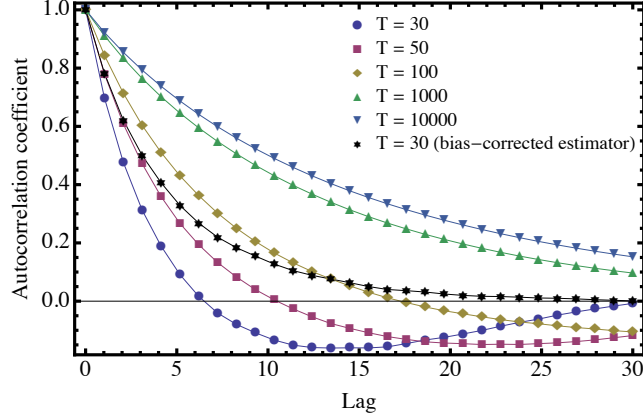


Figure 12: Autocorrelation function of the Laplace diffusion with parameters $\sigma = 0.057$, $m = 0$ and $D = 0.001$ as a function of time series length T . Autocorrelation coefficients are calculated using the mean of each time series and are averaged over 1000 realizations of the process. For comparison, we also show the bias-corrected estimate that emerges when m replaces the time series mean in the autocovariance function (black stars).

Simulations of the diffusion process (9) reproduce the stationary density and autocorrelation profile on a sufficiently long time scale, as illustrated in Figure 11. To demonstrate the relevance of the negative bias in the estimated autocorrelation function for shorter time scales, we have done Monte Carlo simulations of the Laplace diffusion in equation (9) with different time series length T . We then calculated the autocovariance function using the estimator suggested in equation (7). Figure 12 shows that subtracting the time series mean leads to a significant negative bias in the estimated autocorrelation function which becomes smaller if the length of the time series increases. In fact, for short time series as in our data ($T \approx 30$), this particular shape of the autocorrelation function results from an exponentially decaying autocorrelation function and a bias which is linear in the time lag τ . Subtracting m instead of the sample mean of each time series leads to a considerable reduction of the bias, without eliminating it completely. Thus, we also computed the correlation time based on the analytical solution of the autocorrelation function which lead to more accurate estimates of profit persistence if one subscribes to the model (9).

C Regression results

Table 4: Estimates for the slope coefficient in linear ordinary least squares regressions of the location parameter m on the logarithm of firm size. Standard errors are shown in parantheses. Stars indicate statistical significance at the 1 (**), 5 (**), and 10 (*) percent level, respectively.

Size measure	Slope including first bin	Slope excluding first bin
Profit rates		
Sales	0.0054** (0.0020)	0.0021 (0.0019)
Total assets	−0.0011 (0.0017)	−0.0030 (0.0019)
No. of employees	0.0072** (0.0025)	0.0045 (0.0029)
Market value	0.0055*** (0.0016)	0.0042* (0.0019)
Growth rates		
Sales	−0.0010 (0.0012)	−0.0018 (0.0015)
Total assets	−0.0002 (0.0016)	−0.0016 (0.0020)
No. of employees	0.0013 (0.0008)	0.0009 (0.0011)
Market value	0.0107*** (0.0029)	0.0054** (0.0021)

Table 5: Estimates for the slope coefficient in linear ordinary least squares regressions of the logarithm of the dispersion parameter σ on the logarithm of firm size. Standard errors are shown in parantheses. Stars indicate statistical significance at the 1 (**), 5 (**), and 10 (*) percent level, respectively.

Size measure	Slope including first bin	Slope excluding first bin
Profit rates		
Sales	−0.0875** (0.0334)	−0.0210 (0.0184)
Total assets	−0.1015** (0.0303)	−0.0506* (0.0232)
No. of employees	−0.0831** (0.0301)	−0.0267 (0.0198)
Market value	−0.0602 (0.0361)	−0.0200 (0.0412)
Growth rates		
Sales	−0.1044*** (0.0165)	−0.0821*** (0.0175)
Total assets	−0.0634*** (0.0125)	−0.0525*** (0.0149)
No. of employees	−0.0979*** (0.0189)	−0.0721*** (0.0196)
Market value	−0.0913*** (0.0154)	−0.0737*** (0.0174)