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Abstract

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Keywords: education, efficiency, Malmquist-Luenberger, TIMSS

JEL classification: C61, H52, I21

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1. Introduction

In a world characterized by rapid technological change and the importance of innovation processes, the level of academic attainment that students can achieve is essential to improving the levels of wealth and welfare of the citizens in their countries. In the field of public policy in education it is therefore unsurprising to see a growing concern about the assessment of student learning (Denvir and Brown, 1986; Ercikan, 2006). Understanding educational outcomes is critical to effective planning of educational policies, and the assessment of educational reforms.

In this vein, the International Association for the Evaluation of Educational Achievement (IEA) recently published the results of the fifth edition of its Trends in International Mathematics and Science Study (TIMSS). TIMSS 2011 evaluates and describes students' learning in participating countries for these two disciplines, and also provides vital information on other relevant factors (related to curricula, instruction, or availability of resources) that can affect the teaching and learning process.

While the results obtained by a given country in a standardized test (such as TIMSS, or the OECD Programme for International Student Assessment, PISA) are a good reflection of students' academic levels, by themselves they cannot be regarded as a performance indicator for their educational systems and, therefore, their school authorities. The main limitations of these standardized international tests are as follows: (i) the assessment of an organization performance (in this particular case, a country) does not depend exclusively on outcome variables; instead, we can consider efficiency indicators that measure different aspects of the educational process; the results achieved (output) during this process are a consequence of the resources used, the process itself, and environmental variables beyond educational authorities control (Teddlie and Reynolds, 2000); (ii) for a given country, the measure of the results of the educational process should not be constrained to the knowledge students acquire at school, but should also include other outcomes such as the percentage of students failing to meet minimum learning standards (an *undesirable* outcome of the educational process, in terms of educational inequality); and (iii) when measuring students educational achievements at a given point in time, it is difficult to disentangle how much achievement is attributable to the student herself, to her family, or to the strategies applied by previous educational authorities.

To our knowledge, few studies have compared the performance of educational systems in different countries, and there are no previous studies explicitly analyzing how performance has changed over time, and the components of performance. The few studies that have partially addressed these issues include Giménez et al. (2007), who performed a cross-country analysis using Data Envelopment Analysis (DEA) to analyze the efficiency and maximum potential output of educational systems for 31 countries with data from TIMSS 1999. Thieme et al. (2012) carried out a similar comparison for the 54 countries participating in PISA 2006, addressing the first two limitations stated in the preceding paragraph; specifically, these authors apply directional distance functions (DDF) to evaluate efficiency indicators that relate outcome variables with resource variables used in the educational process. The authors jointly evaluate *good* (or desirable) outputs of academic achievement and *bad* (or undesirable) outputs arising from educational inequality. Their results show that it is feasible for a higher education system to combine high levels of student learning and, simultaneously, obtain low inequality levels; however, they found that in most instances both dimensions required significant improvements.

However, to obtain a fuller evaluation of educational systems' performance it would be desirable to evaluate the *change* in performance over time—which, as suggested above, could constitute a third limitation of previous research initiatives. Measuring this change is critical, since there is a general consensus that not only students' achievement needs to be measured, but also their progress, and how much of this progress is attributable to the educational system itself or to external factors. This particular research area in education economics refers to these measures as *growth* studies, which require at least two evaluations at different points in time.

Therefore, in accordance with the rationale presented above, desirable properties of a good education system would include not only its ability to obtain high average academic achievement among its students, but also to ensure that all its students make progress. To achieve this, strategies must be developed that enable relatively disadvantaged students to also make progress and reach basic standards. Therefore, an educational system that evolves satisfactorily will be one that improves the *average* student academic achievement while simultaneously minimizing the percentage of students who do not attain the most basic standards of learning. Similarly, changes in the endowment of resources used by the system will indicate whether the changes in the level of educational achievement (either positive or negative) are due to *technical change*, which might be attributable to an improvement in the educational resources available, or to enhanced *efficiency* when utilizing these resources.

To explore these issues, researchers have proposed a variety of measures to evaluate performance change over time (either due to efficiency change or technical change). Most of these studies follow Färe et al. (1994), although related proposals (closer to the ones we will consider here) have also been developed, including Chung et al. (1997), Pastor and Lovell

(2005), or Luenberger (1992), among others. To measure changes in performance (to achieve educational objectives), in this study we model both good and bad outputs, using the global Malmquist-Luenberger productivity index (hereafter, GML index), developed by Oh (2010). This index, based on contributions by Luenberger (1992) and Pastor and Lovell (2005), successfully remedies some of the weaknesses of the Malmquist-Luenberger index (ML index), by solving the problem of infeasibility of linear programming problems when measuring various cross-sectionals periods using directional distance functions (DDFs).

The GML index is used to measure performance change in the educational systems of 28 countries participating in TIMSS 2007 and 2011 for eighth grade students of basic education in the discipline of mathematics. The results can be interpreted from a multiplicity of angles. They can be exploited from a perspective of orientation (good and bad outputs, good outputs, or bad outputs) or by evaluating the decomposition of the global Malmquist-Luenberger productivity index into its two components—best practice gap change and efficiency change. In general (on average), results indicate a deterioration in educational performance between 2007 and 2011, mainly driven by a decline in the average best practice gap, for both the good and bad output orientation, as well as the good output orientation. In the case of the bad output orientation, educational performance actually improved slightly, also due to an improvement in average best practice gap. We labeled this the *bipartite* decomposition of educational performance, and the ensuing analysis of how the underlying distributions evolved indicated remarkable disparities at country level. The countries participating in the study therefore not only chose different paths to improve their educational performance (which we also describe), but results also varied remarkably among them.

The paper is organized as follows. After this introduction, Section 2 describes the methodological aspects of the global Malmquist-Luenberger Index (GML) and its decomposition to evaluate the performance of education systems over time. The data used for the analysis of educational systems is presented in Section 3. The main results are presented in Section 4, and Section 5 outlines the principal conclusions.

2. Methodology

2.1. Modeling educational performance dynamics

Dynamic efficiency studies often apply the Malmquist index (Caves et al., 1982). This index is used to explain the change in total factor productivity as a result of the change in efficiency or catch-up and technological change. Chung et al. (1997) modified the Malmquist index to apply it to the case of directional distance functions (DDF). These have been widely used in studies measuring efficiency incorporating the environmental impact of the units analyzed by considering the bad outputs of the production process (Sueyoshi and Goto, 2011; Watanabe and Tanaka, 2007; Färe et al., 2005). The new index was named the Malmquist-Luenberger Index.

However, both indices suffer from two problems (Pastor and Lovell, 2005; Oh, 2010). First, circularity is not assured. This property refers to the fact that the change in productivity over a period can be explained by the product of changes in productivity in the different sub-periods within it. Secondly, there is a possibility of infeasibilities in the calculation of the cross-distance functions necessary to calculate them. Although it is necessary and sufficient condition that technical change is Hicks-neutral to ensure circularity (Balk, 2001) and a particular data structure must ensure the absence of feasibility problems (Xue and Harker, 2002), it is often difficult to comply with these conditions in empirical applications. To remedy these two deficiencies, Pastor and Lovell (2005) proposed a modification of the Malmquist index known as the global Malmquist index. Similarly Oh (2010) adapted the Malmquist-Luenberger index to achieve the same properties, leading to the global Malmquist-Luenberger index (hereafter GML).

This paper uses the global Malmquist-Luenberger index proposed by Oh (2010) for the dynamic analysis of the results obtained by countries educational systems. The reason for the choice of this index is that, apart from its desirable properties, it incorporates bad outputs, which educational systems should minimize while maximizing the outputs (good outputs). This index is therefore particularly appropriate in the specific context of education.

Let K be the countries with available information on their educational systems for the years $t = 1 \ldots T$ on M *good* outputs produced, the H *bad* outputs generated from the consumption of N inputs. The production possibility set, is defined by:

$$P(x) = \{(y, b) | x \text{ can produce } (y, b)\} \tag{1}$$

The technology described in Equation (1) must meet the classical axioms proposed by production theory. See, for instance, Färe et al. (2007) for more details.

The efficiency for a given unit belonging to $P(x)$ can be measured by the following directional distance function (Luenberger, 1992; Sueyoshi and Goto, 2011; Oh, 2010):

$$D\left(x, y, b\right) \;=\; \max\left(\beta \;\mid\; \left(y + \beta g_y,\; b - \beta g_b\right) \;\in\; P(x)\right) \tag{2}$$

The DDF in Equation (2) above determines the maximum simultaneously attainable increase and decrease (β) in the good and bad of the output over the vector $g = \left(g_y, g_b\right)$, which defines the desirable directions for improvement both types of outputs. In this paper the vector of $M + H$ components $g = (y,\ b)$ is used as suggested by Chung et al. (1997) and Oh (2010).

The GML index for years t and $t+1$ is defined as follows:

$$GML^{t,t+1}\left(x^t, y^t, b^t, x^{t+1}, y^{t+1}, b^{t+1}\right) = \frac{1 + D^G\left(x^t, y^t, b^t\right)}{1 + D^G\left(x^{t+1}, y^{t+1}, b^{t+1}\right)} \tag{3}$$

where $D^G\left(x, y, b\right) = \max\left(\beta \mid \left(y + \beta g_y,\ b - \beta g_b\right) \in P^G(x)\right)$ is the DDF defined on the global set of production possibilities $P^G(x)$, that is, the set generated by considering all the observations for t and $t+1$. A value greater than one for $GML^{t,t+1}$ indicates improvement in productivity between t and $t+1$, since the distance to the global frontier was greater in t than in $t+1$. A value less than the unit is interpreted as a deterioration.

Expression (3) can be decomposed as follows (Oh, 2010):

$$\begin{aligned} &GML^{t,\,t+1}\left(x^t,\ y^t,\ b^t,\ x^{t+1},\ y^{t+1},\ b^{t+1}\right) = \tfrac{1+D^G\left(x^t, y^t,\ b^t\right)}{1+D^G\left(x^{t+1},\ y^{t+1},\ b^{t+1}\right)} = \\ &= \tfrac{1+D^t\left(x^t, y^t,\ b^t\right)}{1+D^{t+1}\left(x^{t+1},\ y^{t+1},\ b^{t+1}\right)}\ x\ \left[\frac{1 + D^G\left(x^t,\ y^t,\ b^t\right) \big/ 1 + D^t\left(x^t,\ y^t,\ b^t\right)}{1 + D^G\left(x^{t+1},\ y^{t+1},\ b^{t+1}\right) \big/ 1 + D^{t+1}\left(x^{t+1},\ y^{t+1},\ b^{t+1}\right)}\right] = \\ &= \tfrac{TE^{t+1}}{TE^t}\ x\ \left[\tfrac{BPG_{t+1}^{t,\,t+1}}{BPG_t^{t,\,t+1}}\right] = EC^{t,\,t+1}\ x\ BPC^{t,\,t+1} \end{aligned} \tag{4}$$

where $EC^{t,\,t+1}$ reflects the change in technical efficiency or *catching-up* between year t and year $t+1$. If $EC^{t,\,t+1} > 1$, technical efficiency improved in the period. In other words, the unit is closer to its contemporary frontier in year $t+1$ than in t. A value less than unity is interpreted inversely. The term $BPC^{t,\,t+1}$ is a measure of technological change in the period, that is, of how contemporary frontiers have shifted in the period. Expression (4) shows that the expression for the $BPC^{t,\,t+1}$ calculation is:

$$BPC^{t,\,t+1} = \frac{BPG_{t+1}^{t,\,t+1}}{BPG_t^{t,\,t+1}} = \frac{1 + D^G\left(x^t,\ y^t,\ b^t\right) \big/ 1 + D^t\left(x^t,\ y^t,\ b^t\right)}{1 + D^G\left(x^{t+1},\ y^{t+1},\ b^{t+1}\right) \big/ 1 + D^{t+1}\left(x^{t+1},\ y^{t+1},\ b^{t+1}\right)} \tag{5}$$

where:

$$BPG_{t+1}^{t,\,t+1} = \frac{1}{1 + D^G\left(x^{t+1},\ y^{t+1},\ b^{t+1}\right) \big/ 1 + D^{t+1}\left(x^{t+1},\ y^{t+1},\ b^{t+1}\right)} \tag{6}$$

Expression (6) shows that $BPG_{t+1}^{t,\,t+1}$ is the inverse of the ratio between the distance to the global frontier defined by $P^G\,(x)$ and the contemporary frontier defined by $P\,(x)$ at time $t+1$. If $BPC^{t,t+1} > 1$, the contemporary frontier in $t+1$ is closer to the global frontier than in t, and therefore technological progress was made. If $BPC^{t,t+1} < 1$ there was no technological progress.

Various methods can be used to calculate $D^G\left(x^t, y^t, b^t\right)$ and $D^t\left(x^t, y^t, b^t\right)$. In this paper we use Data Envelopment Analysis models (Charnes et al., 1978), which have been widely used in efficiency studies (for an extensive review of their use in the literature, see Emrouznejad et al., 2008). To calculate of $D^t\left(x^t, y^t, b^t\right)$ we consider the following linear program (under the assumption $g = (y, b)$) for each country analyzed (Mandal and Madheswaran, 2010):

$$
\begin{aligned}
& Max\ \beta \\
& s.t \\
& \textstyle\sum_{k=1}^{K} \lambda_k\, y_{km}^{t} \;\geq\; y_m^{ot}\,(1+\beta) \quad m = 1 \ldots M \\
& \textstyle\sum_{k=1}^{K} \lambda_k\, b_{kh}^{t} \;\leq\; b_h^{ot}\,(1-\beta) \quad h = 1 \ldots H \\
& \textstyle\sum_{k=1}^{K} \lambda_k\, x_{kn}^{t} \;\leq\; x_n^{ot} \quad n = 1 \ldots N \\
& \textstyle\sum_{k=1}^{K} \lambda_k \;=\; 1 \\
& \beta \geq 0\ ;\ \lambda_k \geq 0 \quad k = 1 \ldots K
\end{aligned}
\tag{7}
$$

where β is the maximum achievable increase and decrease in both good and bad outputs, respectively, y_{km}^t represents the output m of the unit k in year t, b_{kh}^t is the bad output h of the unit or country k in year t, and x_{kn}^t is the input n used by the country's education system k in year t. The observed levels of good outputs, bad outputs and inputs for the evaluated country in year t are represented by y_m^{ot}, b_h^{ot} and x_{kn}^{ot}, respectively.

Analogously, the following linear program must be solved to calculate $D^G\left(x^t,\ y^t,\ b^t\right)$:

$$
\begin{aligned}
& \text{Max } \beta \\
& s.t \\
& \textstyle\sum_{k=1}^{K} \sum_{T=t}^{t+1} \lambda_k^t\, y_{km}^{T} \;\geq\; y_m^{ot}\,(1+\beta) \quad m = 1 \ldots M \\
& \textstyle\sum_{k=1}^{K} \sum_{T=t}^{t+1} \lambda_k^t\, b_{kh}^{T} \;\leq\; b_h^{ot}\,(1-\beta) \quad h = 1 \ldots H \\
& \textstyle\sum_{k=1}^{K} \sum_{T=t}^{t+1} \lambda_k^t\, x_{kn}^{T} \;\leq\; x_n^{ot} \quad n = 1 \ldots N \\
& \textstyle\sum_{k=1}^{K} \sum_{T=t}^{t+1} \lambda_k^T \;=\; 1 \\
& \beta \geq 0\ ;\ \lambda_k^T \geq 0 \quad k = 1 \ldots K
\end{aligned}
\tag{8}
$$

In this study, apart from assuming the directional vector $g = (y, b)$ to calculate the GML, two alternative directional vectors are explored: $g = (y, 0)$ and $g = (0, b)$. These additional calculations quantify productivity changes in two new directions: one prioritizing increases only in the *good* outputs and other prioritizing decreasing only in the *bad* outputs. This information is useful to test whether countries have moved toward the possible achievement of higher *potential* earnings in the period analyzed. Other directional vectors could also have been explored, as none of the explored vectors might be representing the optimal movement toward the frontier. However, it is relevant to ascertain whether an educational system should aim to improve *both* types of outputs simultaneously, or only one of them. This question should be considered in the definition of any educational policy.

Various approaches to integrate the undesirable outputs in the efficiency estimations can be found in the extant literature. The most popular approach is probably to consider the bad outputs as *weakly disposable* (basically modifying the restrictions in order to accept proportional reductions in the bad as well as in the good outputs). For more details on this option see Färe et al. (1989) and Färe and Grosskopf (2004). However, the debate on the problems and the solutions of this option is far from over; see, for instance, Kuosmanen (2005), Kuosmanen and Podinovski (2009), Färe and Grosskopf (2009), or Picazo-Tadeo and Prior (2009), among others. Another possibility is to convert the undesirable bad outputs into *desirable* (i.e. strongly disposable) good outputs, as suggested by Golany and Roll (1989) and Seiford and Zhu (2002), but this conversion may influence significant changes in the level of efficiency found. Finally, following Reinhard et al. (2002) and Hailu and Veeman (2001), perhaps the most intuitive option is to consider the bad outputs as strongly disposable inputs. Because of its simplicity, this option was selected in our proposal.

2.2. Bipartite decomposition of the relative contributions to educational performance

In accordance with the expressions detailed in the previous section, the global Malmquist-Luenberger (GML) index is decomposed into technical change (EC) and best practice gap change (BPC). Apart from analyzing how the different components contribute to the overall change of GML *on average*, we can also consider a distribution dynamics approach to analyze what the largest contributors to the variation in performance are, as measured by GML between periods t (2007) and $t+1$ (2011). To this end, we use nonparametric density estimation, based on kernel smoothing.

We rewrite expression (3) above as follows:

$$gml^{EC\times BPC} = EC^{t,t+1} \times BPC^{t,t+1} \tag{9}$$

according to which we use expression $gml^{EC\times BPC}$ to indicate that the change in educational achievement is obtained by successively multiplying its three components. This in turn, allows us to construct counterfactual distributions by sequentially introducing each of the factors. Specifically, the counterfactual educational achievement change attributable to changes in efficiency would be:

$$gml^{EC} = EC^{t,t+1} \tag{10}$$

which isolates the effect on the distribution of changes due to efficiency only, assuming BPC does not contribute to the change in educational achievement (gml).

Analogously, and if we extend this sequential decomposition, we proceed as follows:

$$\begin{aligned} gml^{EC\times BPC} &= EC^{t,t+1} \times BPC^{t,t+1} \\ &= gml^{EC} \times BPC^{t,t+1} \end{aligned} \tag{11}$$

We refer to the decomposition in (11) as the <u>bipartite decomposition</u> of the <u>relative contributions to the changes in the distribution of educational performance</u>.

3. Data, inputs and outputs

This study uses information from the education systems of 28 countries participating in the TIMSS 2007 and 2011. TIMSS stands for *Trends in International Mathematics and Science Study*, and is carried out by the International Association for the Evaluation of Educational Achievement (IEA). Its purpose is to measure students' learning achievement at the end of fourth and eighth grades in basic mathematics and science. The study is performed every four years, and the 2011 edition was the fifth version of the study. Its design allows for comparisons *over time* and *across countries* participating in the study. In this study, we consider information corresponding to eighth grade mathematics students. For this particular grade, 47 countries participated in the 2011 study, and 50 in 2007.

The sample of schools and students is selected by the IEA at country level, representing each of the grades under analysis. In general, for every country in the sample there are approximately 4,000 students from 150 to 200 schools, for each of the assessed grades. Additionally, each country can apply for a larger sample size should it be interested in a particular type of segmentation (by type of administration, location, etc.). The TIMSS also collects information on principals, teachers and students, which enables a framework to be set to analyze the results corresponding to the learning process. Because this evaluation framework is agreed among the countries participating in the study, it does not necessarily compile the same information from participants in each version of the study.

As noted previously, the methodology described in the preceding section is used to evaluate the *change* in the performance of educational systems for achieving their goals. To this end, following the extant literature (Carlson, 2001) we consider that a good education system is not only one in which students obtain a high degree of *average* academic achievement for their students but it also ensures that all students make *progress*. For this to occur, strategies must also be developed to enable more disadvantaged students to make progress and achieve basic educational levels. Therefore, an educational system that makes satisfactory progress will enable not only an *average* improvement in its students' academic achievement, but will also *minimize* the percentage of students who do not achieve basic standards of learning. Analogously, changes in the resource endowments used will indicate whether the changes in the achievement of the educational goals (either positive or negative) are due to *technical change* (which might result from higher endowments of educational resources) or to improved *efficiency* in their usage.

The TIMSS reports provide standardized information on both educational outcomes and resources used for this purpose in at least two points in time. For this particular study we used the reports corresponding to years 2007 and 2011. The learning outcomes in mathematics and science are reported by TIMSS in two ways for each participating country: (i) on a scale ranging from 0 to 1,000, with a standardized international average of 500 points, and a standard deviation of 100 points. This average corresponds to the set of countries participating in the first edition of TIMSS in 1995, and was set as the benchmark for comparability between years; (ii) for four performance levels describing the learning levels students achieved. The *advanced* level corresponds to students with more than 625 points, the *high* level corresponds to students with between 550 and 624 points, the *intermediate* level corresponds to students with between 475 and 549 points, and the *low* or basic level corresponds to students with between 400 and 474 points. Analogously, we report the percentage of students who fail to achieve this basic standard and, therefore, fall outside the range.

Accordingly, our definition of good output (y_1) was the average achievement of each country for its eighth year basic education students in mathematics and as bad output (y_2) the percentage of students who did not reach the basic standards (i.e., whose achievement was too low for estimation), corresponding to the same grade and discipline. For both indicators we have data for 2007 (time t) and 2011 (time $t+1$). The average values, corresponding to both outputs for each country for years 2007 and 2011, are reported in the first four columns of Table 1.

As a reference, in the eighth year of basic (primary) education of TIMSS 2011, only 14 countries (out of 42) obtained scores higher than 500 in the TIMSS scale, whereas in 2007 only 12 countries (out of 50) had scores higher than 500. In 2011 the highest average score for mathematics at primary education level corresponded to the Republic of South Korea (613 points), followed by Singapore (611 points), Chinese Taipei (609 points) and Hong Kong (586 points). In 2007, the highest academic achievements were obtained by Chinese Taipei (598 points), followed by the Republic of South Korea (597 points), Singapore (593 points), Hong Kong (572 points) and Japan (570 points). Out of the 28 countries participating in both years, the highest progress corresponded to the Palestinian National Authority (37 point increase), followed by Italy (35 point increase). In contrast, the largest decline corresponded to Malaysia (34 point decrease between the two years), followed by Jordan (21 point decline).

Meanwhile, in eighth grade mathematics for 2011, the average distribution of participating countries indicates that 18% of students did not reach the minimum standards, 21% reached the lowest level, 30% the intermediate level, 22% the high level, and only 9% the most advanced level. The percentage of students not reaching the minimum standards is very heterogeneous across countries. Among the 10 countries with the highest scores, this percentage is 2% (on average), whereas for the 10 countries with the lowest scores the percentage is 55%.

In our sample, the sharpest declines in the percentage of lowest-achieving students correspond to Italy, Georgia and the Palestinian National Authority; this percentage fell by 10% between 2007 and 2011 in these countries. In contrast, Malaysia was the country with the highest increase in the percentage of students below the minimum standards (from 50% to 64%).

The inputs of the model correspond to two variables for which there was available information for both years, namely, learning hours in mathematics during the academic year (x_1), and teacher quality, measured as the percentage of students whose teachers consider themselves to be "very well" trained for teaching mathematics (x_2). The average values corresponding to both inputs, for each country in our sample, and for years 2007 and 2011 are reported in the last four columns of Table 1.

The hours corresponding to eighth grade mathematics range from 76 hours (Syria) to 158 hours (Chinese Taipei) for the year 2007, and from 97 hours (Sweden) to 173 hours (Indonesia) for 2011. The sample average is 135 hours for 2011 and 120 hours for 2007. The country with the greatest increase in the number of teaching hours is Bahrain (by 46 hours), followed by Syria (by 42 hours). Teaching time fell in only two countries: Jordan (by 11 hours) and Hong Kong (by 10 hours).

The data indicate that the best teaching quality corresponds to teachers in the United States, England, Georgia and Romania, with 94% of students whose teachers consider themselves to be "very well" prepared for teaching mathematics. The lowest value for this resource corresponds to Indonesia (54%), followed by Thailand (55%). The country with the greatest improvement in this indicator was Lithuania (23% increase), followed by Japan (16% increase). In contrast, the country with the sharpest decline in teaching quality was Indonesia (27% decrease), followed by Ukraine (18% decrease).

The average values corresponding to the inputs and outputs of the model for each country in eighth grade of mathematics for years 2007 and 2011 used in the analysis of the global Malmquist-Luenberger index are reported in Table 1.

4. Results

4.1. Efficiency change, best practice gap change, and performance change: analysis based on summary statistics

Tables 2, 3 and 4 show the results of the global Malmquist-Luenberger index and its decomposition for the countries in our sample and for the different directions selected, that is, good and bad output orientation (Table 2), good output orientation (Table 3), and bad output orientation (Table 4).

Results vary remarkably in two dimensions, namely, for the two components of the global Malmquist-Luenberger index (efficiency change and best practice gap), and for the different orientations chosen—good and bad output orientation, good output orientation, and bad output orientation.

A comparison of the bottom three rows in Tables 2, 3, and 4 shows clear differences in both dimensions of variability. On average (for both the arithmetic and geometric means), the *GML* index shows a *mean* deterioration in educational performance when considering either the *good and bad* output orientation, and the *good* output orientation (Tables 2 and 3) for the countries in our sample. However, for the *bad* output orientation (Table 4), *on average*, there is virtually no change in educational performance—the arithmetic mean is 1.0030 and the geometric mean is 1.0008.

An analysis of the components of the GML index, that is, the efficiency change (EC) and best practice gap (BPC), also provides different results for, on the one hand, good and bad output orientation and good output orientation and, on the other hand, bad output orientation. Whereas in the former case efficiency change (EC) contributes *positively* to overall performance change, in the latter case the contribution is *negative*. In contrast, the best practice gap (BPC) shows an opposite result—the contribution is negative in the case of good output and good and bad output orientation, and positive in the case of bad output orientation.

However, these are *average* results, concealing very heterogeneous findings at the country level. This is partly shown by the standard deviation values, which are particularly high for the efficiency change in the case of the good and bad and good output orientations (Tables 2 and 3), with values of 0.1706 and 0.1851 (see the first columns in Tables 2 and 3, respectively). In contrast, for the same orientations, the dispersion for the best practice gap is much lower, with values of 0.0678 and 0.0613 (see the second columns in Tables 2 and 3, respectively). Combining both components of performance change, the standard deviation for each orientation is 0.1145 and 0.1272 (see the third column in Tables 2 and 3, respectively).

In the case of bad output orientation (Table 4), there is a greater balance, in terms of dispersion across countries, between the two components of performance change (0.0610 and 0.0625 for EC and BPC, respectively), resulting in a value of 0.0674 value corresponding to the standard deviation for the global Malmquist-Luenberger index.

However, despite the more moderate dispersion values found for both EC and GML in the case of the bad output orientation (compared to the other two orientations), results differ remarkably for some specific countries. For instance, as indicated in the third column of Table 4, (positive) change in performance was substantial in Chinese Taipei and Singapore; however, the reasons were not coincidental—for Chinese Taipei it was mostly due to efficiency change ($EC^{t,t+1}_{\text{Chinese Taipei}} = 1.2845$), whereas for Singapore best practice gap change played a major role ($BPC^{t,t+1}_{\text{Singapore}} = 1.1591$). In contrast, Japan experienced a remarkable deterioration in educational performance ($GML^{t,t+1}_{\text{Japan}} = 0.7671$) due to best practice gap decline ($BPC^{t,t+1}_{\text{Japan}} = 0.7671$), whereas efficiency stagnated ($EC^{t,t+1}_{\text{Japan}} = 1.0000$).

This multiplicity of different cases is even higher when analysis focuses on good and bad output orientation (Table 2) and good output orientation (Table 3), although in the particular case of bad output orientation virtually *all* countries showed best practice decline ($BPC^{t,t+} < 1$), with the exception of Korea ($BPC^{t,t+}_{\text{Korea}} = 1$, see Table 3).

The variety of paths along which countries' educational performance evolves cannot easily be summarized by only two statistics (standard deviation and mean, either arithmetic or geometric), which makes it difficult to explore results in detail. An informative complement consists of applying the bipartite decomposition of performance change based on kernel density estimation proposed in Section 2.2, as we shall see below.

4.2. Classifying educational systems: should we stress excellence or reduce inequalities?

Table 5 shows a classification of the countries evaluated considering both the results of the GML index and the assessment orientation. The first group of countries ($G1$), composed of Italy and Singapore, are countries that improved in the period under any orientation. We refer to this group as "overall improvement". The second group ($G2$) comprises countries with a positive evolution in the period when only the bad output orientation is considered. They are therefore countries that appear to have strived to minimize the number of students below acceptable levels and, consequently, they have focused on reducing inequality in their educational systems. This group is labeled "inequality improvement" and is composed of Australia, Bahrain, Chinese Taipei, Georgia, Hong Kong, Iran, Norway, Oman, Palestinian, Slovenia, Tunisia and United States.

The next group ($G3$) comprises Jordan and Indonesia. The results for these countries suggest that they mainly focused on improving the average performance of their students over the 2007–2011 period; we can draw this conclusion from the favorable evaluation they obtain only when the evaluation is under an orientation that prioritizes good outputs. This group is termed "average achievement". The fourth group ($G4$), named "simultaneous improvement", comprises two countries—Ukraine and Ghana—whose results suggest that they directed their efforts toward improving both average achievement and inequality simultaneously.

The fifth group ($G5$) contains only one country—South Korea—that maintained high levels of efficiency in both periods and is termed "stable". Finally, group ($G6$) consists of countries whose performance declined during the analyzed period, regardless of their orientation; countries in this group, termed "Decline", are England, Hungary, Lithuania, Malaysia, Romania, Sweden, Syrian and Thailand.

4.3. Bipartite decomposition of performance change

The results of the analysis proposed in Section 2.2 are reported in Figures 1, 2, 3 and 4. Each figure is divided into three panels. The upper panels correspond to the analysis of performance change considering a good and bad output orientation; the central panel reports results for the good output orientation; and the lower panel refers to bad output orientation. Each panel is also divided in two sub-figures to provide a sequential analysis of the contribution of each performance change component. The sequential order is shown in both directions (Figure 1 vs. 2, and Figure 3 vs. 4). The vertical line in each figure corresponds to the (arithmetic) mean of the underlying density.

Given some of the particularities of the data used, the densities were also estimated for different values of the smoothing parameter, or bandwidth (h), which tunes the amount of *bumps* under each curve—higher values of h tend to smooth more, revealing fewer data particularities, low values of h tend to smooth less, providing more detail but generating (in some cases) fuzzy graphics. Specifically, Figures 1 and 2 report results for a *global* bandwidth (the amount of smoothing is the same at all data points), for which we followed the proposals of Sheather and Jones (1991). In the case of Figures 3 and 4, the amount of smoothing varies *locally*, depending on the structure of the data at a given point, for which we followed Loader (1996).

The analysis in the upper panel of Figure 1 shows that when considering a good and bad output orientation, the contribution of efficiency to the change of the global Malmquist-Luenberger index is very heterogeneous, as indicated by several bumps shown by density corresponding to gml^{EC} (Figure 1.a). However, the contribution of the best practice change, shown in Figure 1.b, offsets the heterogeneity of gml^{EC}, leading to a much smoother density when the two effects are combined ($gml^{EC\times BPC}$). Actually, *on average*, as indicated by the vertical lines in Figures 1.a. and 1.b, although the effect of efficiency change (gml^{EC}) is positive (the solid vertical line is above 1), the contribution of the best practice gap leads to a *negative* combined effect (the dashed vertical line is below 1). The smoother lines depicted when choosing local bandwidths, as shown in Figures 3.a and 3.b point in the same direction, excepting for the bumps corresponding to gml^{EC}, which are smoothed out in Figure 3.a.

These discrepancies are also present when the sequential order is reversed, as shown in the upper panel of Figure 2 (Figures 2.a and 2.b), for the global bandwidth, and the upper panel of Figure 4, for the local bandwidth (Figures 4.a and 4.b). The analysis performed in the reverse order indicates that the discrepancies for gml^{BPC} are even higher than those for gml^{EC}; this is particularly apparent when a global bandwidth is chosen. Therefore, countries follow very different paths to obtain their productivity change index.

Results change when considering either a good output orientation (Figures 1.c and 1.d for the global bandwidth, and Figures 3.c and 3.d for the local bandwidth), or a bad output orientation (Figures 1.e and 1.f for the global bandwidth, and Figures 3.e and 3.f for the local bandwidth). Computations were also performed with the reversed the direction of causality (see lower panels of Figures 2 and 4.

Regarding the good output orientation, when considering a global bandwidth there are remarkable disparities across countries for gml^{BPC} (see Figures 1.c and 1.d) to the point that, on average, the contribution of the best practice gap change is *negative*. In contrast, the probability mass corresponding to the contribution of efficiency change is positive—much of the density is above 1 (see Figures 1.c and 3.c). However, when considering the bad output orientation, discrepancies are remarkable for both components of of the educational performance index. Although according to the local bandwidth figures (Figures 3.e, 3.f, 4.e and 4.f) probability concentrates tightly around unity, this effect is partly derived from the choice of bandwidths, since choosing global bandwidths results in much fuzzier graphics (Figures 1.e, 1.f, 2.e and 2.f).

5. Conclusions

In this paper, we have considered some relatively recent proposals to analyze educational performance and how it changes over time. Specifically, we used the latest release of the tests for analyzing Trends in International Mathematics and Science Study (TIMSS). This initiative provides a framework for evaluating and describing the learning processes of students in participating countries, for the two disciplines analyzed, providing relevant information on additional factors involved in these processes.

We considered the global Malmquist-Luenberger index (GML), which is particularly interesting in the context of education. This index is not only appropriate for its highly desirable properties; it also suits our context because it incorporates bad outputs which, ideally, educational systems should minimize while simultaneously maximizing the outputs—or, more correctly, good/desirable outputs.

The results of the different evaluations of the GML index show, on average, a deterioration in educational systems' performance when both a simultaneous good and bad output orientation is considered, as well as when only a good output orientation is considered. However, assessing performance when adopting only a bad output orientation shows, on average, a slight improvement in performance for the countries in the sample.

Similarly, the average results reveal the variety of emphases followed by the national education systems during the analyzed period (2007–2011). In general (on average) we find a clear preference for technological changes aimed at obtaining higher levels of educational equality, in contrast to an emphasis on efficiency improvements when pursuing academic achievement (excellence) and equality simultaneously (i.e., good and bad output orientation), or academic achievement (good output orientation) exclusively.

This is consistent with: (i) an increasing concern among the countries in the sample to enhance the quality of human capital among the population as equally as possible; (ii) the budgetary constraints that countries faced during most of the analyzed period; and (iii) the different levels in the learning curve of the educational systems with respect to the different objectives pursued (or orientations adopted). Therefore, after years of public policies aimed at achieving improvements in overall academic achievement and equality, or only average academic achievement, the tendency to adopt policies aimed at achieving efficiency improvements is not entirely surprising. This contrasts with the necessary commitment to stress technological improvements when there is no knowledge base to facilitate performance improvements via enhanced efficiency.

Apart from the *average* results, there are remarkable discrepancies in the results that point in two directions. First, there is a striking heterogeneity in the results corresponding to the different components of the GML index with respect to the different emphases (orientations) evaluated. The dispersion of results is especially high among countries for the efficiency change (EC) when adopting good and bad output as well as good output orientations (0.1706 and 0.1851, respectively); however, it is smaller when emphasizing bad outputs (0.0610). Moreover, for the three different orientations the dispersion observed for countries' performance is low regarding the technological change component (BPC). This has a twofold implication: (i) the differences in overall performance (GML) among countries are mainly driven by changes in efficiency rather than technological change; and (ii) changes in equality are more difficult to achieve and probably they require a longer time horizon.

Second, this heterogeneity is also reflected in the ranking of countries. In this classification only two countries (Italy and Singapore) improve their performance in terms of the three possible orientations, or emphases. However, only Singapore adopts this strategy when both efficiency improvements *and* technological change are considered. Italy follows the exclusive strategy of improvements in efficiency. It is also interesting to note that the largest group consists of countries that only improve in the area of equality (bad output orientation). This is consistent with the new challenges facing educational systems. This group is composed of both high and low academic achievement countries; the former include Australia, Chinese Taipei, Hong Kong, Norway, United States and Slovenia, whereas in the latter we would include Bahrain, Georgia, Iran, Oman, Palestinian and Tunisia. This constitutes evidence of the transversality of this priority. Finally, the presence of high academic achievement countries such as England, Hungary and Sweden in the group of countries with performance decline is also noteworthy. However, this outcome is coincidental with TIMSS results indicating that during the evaluation period the academic achievements of all these countries deteriorated while the percentage of students who failed to reach the minimum standards increased.

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Table 1: Descriptive statistics on inputs and outputs based on TIMMS (2007 and 2011)

Country	Good output (y_1)		Bad output (y_2)		Input 1 (x_1)		Input 2 (x_2)	
	Academic achievement, 2007	Academic achievement, 2011	2007	2011	Mathematics (learning hours) 2007	Mathematics (learning hours) 2011	Teaching quality, 2007	Teaching quality, 2011
Australia	496	505	39	37	131	143	91	91
Bahrain	398	409	81	74	96	142	88	88
Chinese Taipei	598	609	14	12	158	166	74	72
England	513	507	31	35	113	116	95	94
Georgia	410	431	74	64	110	123	86	94
Ghana	309	331	96	95	146	165	85	87
Hong Kong (S.A.R.)	572	586	15	11	148	138	67	82
Hungary	517	505	31	35	99	119	89	86
Indonesia	397	386	81	85	136	173	81	54
Iran	403	415	80	74	99	124	78	82
Italy	463	498	46	36	136	155	65	64
Japan	570	570	13	13	105	108	51	67
Jordan	427	406	65	74	141	130	89	84
Korea, Rep. of	597	613	10	7	104	137	70	79
Lithuania	506	502	35	36	116	132	70	93
Malaysia	474	440	50	64	123	123	79	83
Norway	469	475	52	49	113	125	87	85
Oman	372	366	86	84	150	161	84	87
Palestinian Nat. Auth.	367	404	85	75	100	134	86	86
Romania	461	458	54	56	122	145	87	94
Singapore	593	611	12	8	124	138	82	86
Slovenia	501	505	35	33	113	121	79	88
Sweden	491	484	40	43	93	97	79	87
Syrian Arab Rep.	395	380	83	83	76	118	74	79
Thailand	441	427	66	72	124	129	47	55
Tunisia	420	425	79	75	126	131	87	78
Ukraine	462	479	54	47	130	132	90	72
United States	508	509	33	32	148	157	93	94

Table 2: Educational improvement, good and bad output orientation (2007–2011)

Country	Efficiency change $EC^{t,t+1}$	Best practice gap change $BPC^{t,t+1}$	Global Malmquist-Luenberger index $GML^{t,t+1}$
Australia	1.0145	0.9296	0.9430
Bahrain	0.7738	0.9739	0.7536
Chinese Taipei	1.1229	0.9482	1.0647
England	1.0471	0.9194	0.9627
Georgia	1.0225	0.9194	0.9401
Ghana	0.9866	1.0166	1.0029
Hong Kong (S.A.R.)	1.0267	0.9791	1.0052
Hungary	0.8839	0.9194	0.8126
Indonesia	1.5435	0.7627	1.1773
Iran	0.8942	0.9226	0.8250
Italy	1.4351	0.7612	1.0924
Japan	1.0000	0.9337	0.9337
Jordan	1.0643	0.9287	0.9884
Korea, Rep. of	1.0000	1.0000	1.0000
Lithuania	0.9291	0.9194	0.8542
Malaysia	1.0006	0.9194	0.9200
Norway	0.9958	0.9194	0.9156
Oman	0.9884	1.0054	0.9937
Palestinian Nat. Auth.	0.8935	0.9540	0.8524
Romania	0.9092	0.9490	0.8628
Singapore	1.0677	1.0180	1.0870
Slovenia	1.0239	0.9194	0.9413
Sweden	1.0279	0.9194	0.9451
Syrian Arab Rep.	0.6739	0.9204	0.6203
Thailand	1.0870	0.7612	0.8274
Tunisia	1.1030	0.9028	0.9957
Ukraine	1.2631	0.8394	1.0603
United States	1.0360	0.9338	0.9675
Arithmetic mean	1.0291	0.9213	0.9409
Geometric mean	1.0163	0.9187	0.9337
Standard deviation	0.1706	0.0678	0.1145

Table 3: Educational improvement, good output orientation (2007–2011)

Country	Efficiency change $EC^{t,t+1}$	Best practice gap change $BPC^{t,t+1}$	Global Malmquist-Luenberger index $GML^{t,t+1}$
Australia	1.0145	0.9293	0.9427
Bahrain	0.7564	0.9329	0.7057
Chinese Taipei	1.1909	0.9224	1.0984
England	1.0471	0.9194	0.9627
Georgia	1.0225	0.9194	0.9401
Ghana	1.1810	0.8159	0.9636
Hong Kong (S.A.R.)	1.0815	0.9531	1.0308
Hungary	0.8839	0.9194	0.8126
Indonesia	1.6152	0.7612	1.2294
Iran	0.8942	0.9226	0.8250
Italy	1.4351	0.7612	1.0924
Japan	1.0000	0.9337	0.9337
Jordan	1.1081	0.9266	1.0268
Korea, Rep. of	1.0000	1.0000	1.0000
Lithuania	0.9291	0.9194	0.8542
Malaysia	1.0006	0.9194	0.9200
Norway	0.9958	0.9194	0.9156
Oman	1.1071	0.8329	0.9221
Palestinian Nat. Auth.	0.8935	0.9278	0.8290
Romania	0.9092	0.9261	0.8420
Singapore	1.1576	0.9767	1.1306
Slovenia	1.0239	0.9194	0.9413
Sweden	1.0279	0.9194	0.9451
Syrian Arab Rep.	0.6739	0.9204	0.6203
Thailand	1.0870	0.7612	0.8274
Tunisia	1.1030	0.9028	0.9957
Ukraine	1.2631	0.8394	1.0603
United States	1.0507	0.9069	0.9528
Arithmetic mean	1.0519	0.9003	0.9400
Geometric mean	1.0369	0.8982	0.9313
Standard deviation	0.1851	0.0613	0.1272

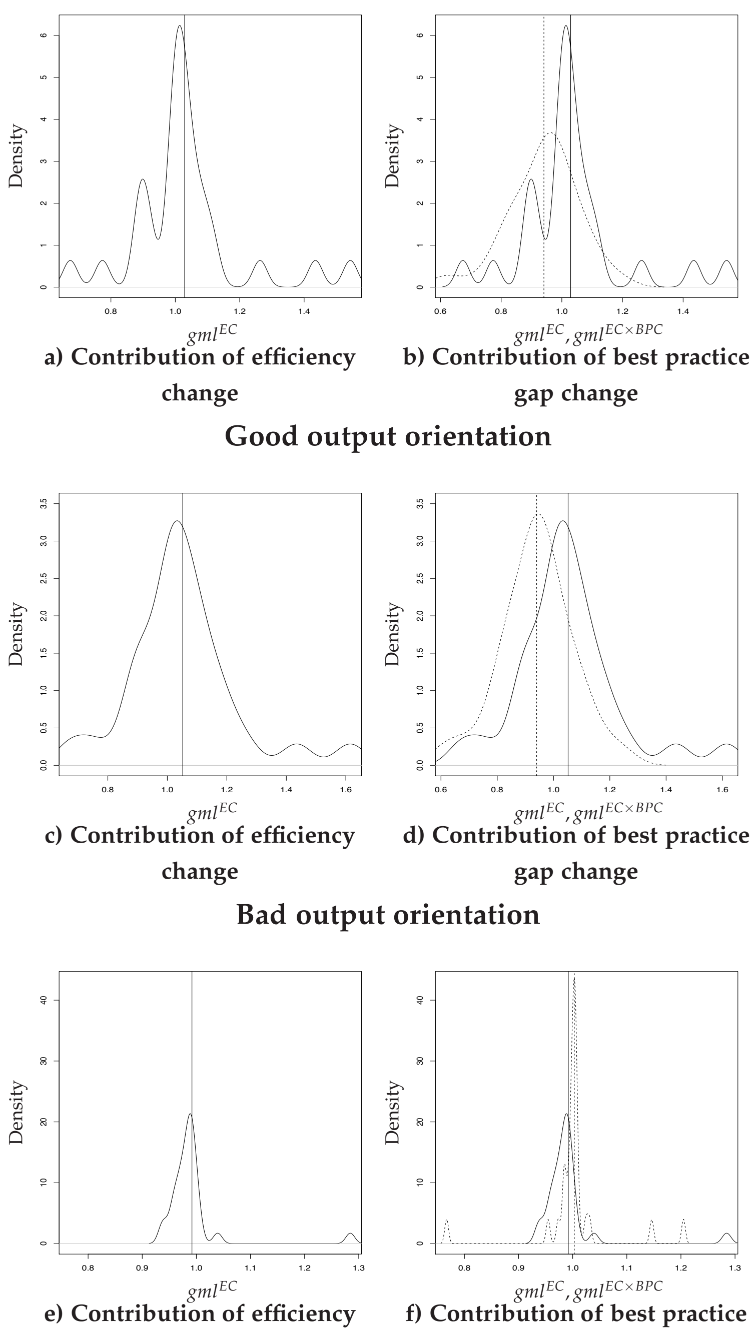
Table 4: Educational improvement, bad output orientation (2007–2011)

Country	Efficiency change $EC^{t,t+1}$	Best practice gap change $BPC^{t,t+1}$	Global Malmquist-Luenberger index $GML^{t,t+1}$
Australia	0.9690	1.0379	1.0058
Bahrain	0.9901	1.0137	1.0036
Chinese Taipei	1.2845	0.8922	1.1460
England	0.9391	1.0479	0.9840
Georgia	0.9917	1.0155	1.0071
Ghana	0.9928	1.0088	1.0015
Hong Kong (S.A.R.)	0.9774	1.0551	1.0312
Hungary	0.9376	1.0181	0.9546
Indonesia	0.9845	1.0136	0.9979
Iran	0.9895	1.0140	1.0034
Italy	0.9963	1.0279	1.0242
Japan	1.0000	0.7671	0.7671
Jordan	0.9755	1.0185	0.9936
Korea, Rep. of	1.0000	1.0000	1.0000
Lithuania	0.9549	1.0438	0.9968
Malaysia	0.9582	1.0274	0.9845
Norway	0.9786	1.0260	1.0041
Oman	0.9884	1.0120	1.0002
Palestinian Nat. Auth.	0.9944	1.0119	1.0063
Romania	0.9740	1.0245	0.9979
Singapore	1.0394	1.1591	1.2048
Slovenia	0.9644	1.0434	1.0062
Sweden	1.0055	0.9790	0.9844
Syrian Arab Rep.	0.9859	1.0049	0.9907
Thailand	0.9692	1.0042	0.9733
Tunisia	0.9874	1.0148	1.0021
Ukraine	0.9857	1.0246	1.0099
United States	0.9581	1.0471	1.0032
Arithmetic mean	0.9919	1.0126	1.0030
Geometric mean	0.9903	1.0106	1.0008
Standard deviation	0.0610	0.0625	0.0674

Table 5: Classification of countries according to their educational achievements

Group ID	Strategy/achievement	Countries in the group
$G1$	Overall improvement	Italy, Singapore
$G2$	Inequality improvement	Australia, Bahrain, Chinese Taipei, Georgia, Hong Kong, Iran, Norway, Oman, Palestinian Nat. Auth., Slovenia, Tunisia, United States
$G3$	Average achievement	Jordan, Indonesia
$G4$	Simultaneous improvement	Ukraine, Ghana
$G5$	Stable	Korea
$G6$	Decline	England, Hungary, Lithuania, Malaysia, Romania, Sweden, Syrian, Thailand

Figure 1: Kernel density plots of the bipartite decomposition of educational improvement, good and bad output orientation



Good and bad output orientation

a) Contribution of efficiency change

b) Contribution of best practice gap change

Good output orientation

c) Contribution of efficiency change

d) Contribution of best practice gap change

Bad output orientation

e) Contribution of efficiency change

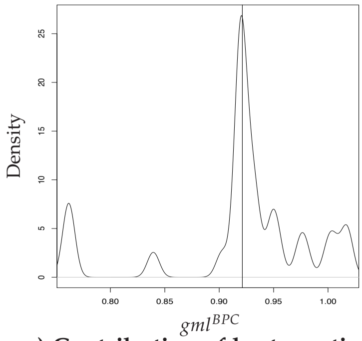
f) Contribution of best practice gap change

gml^{EC} ——— $gml^{EC\times BPC}$ - - - - - -

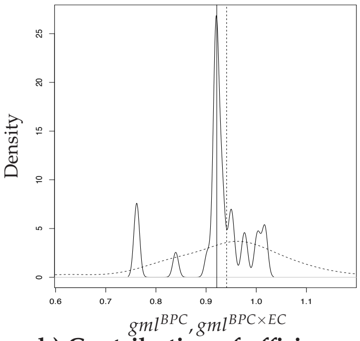
Notes: All figures contain densities estimated using kernel density estimation for the different components of the bipartite decomposition in expression (11), considering the good and bad output orientation. The vertical lines in each plot represent the average for each component of the decomposition. Densities were estimated using a Gaussian kernel and the Sheather and Jones (1991) plug-in bandwidth.

Figure 2: Kernel density plots of the bipartite decomposition of educational improvement, good and bad output orientation

Good and bad output orientation

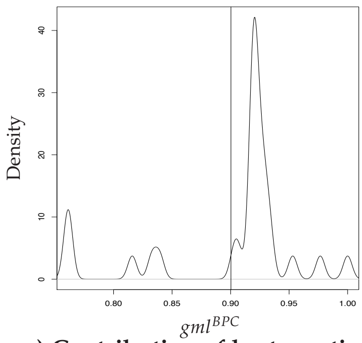


a) Contribution of best practice gap change

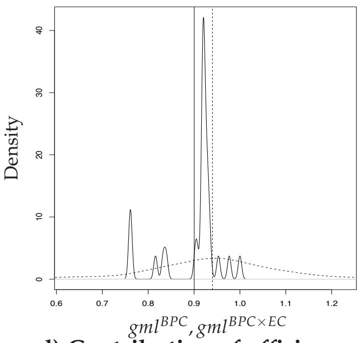


b) Contribution of efficiency change

Good output orientation

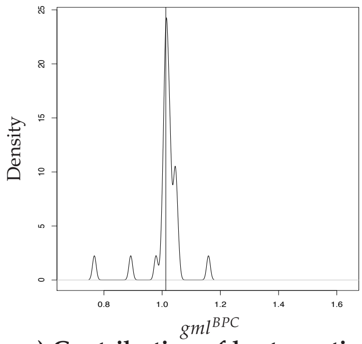


c) Contribution of best practice gap change

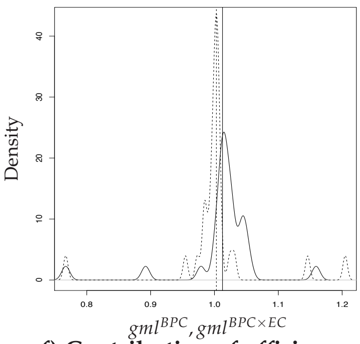


d) Contribution of efficiency change

Bad output orientation



e) Contribution of best practice gap change



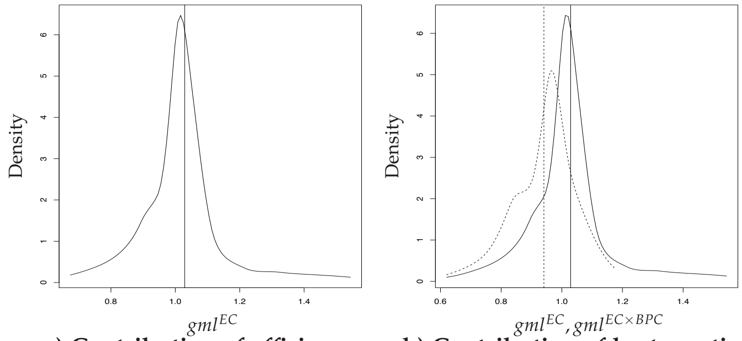
f) Contribution of efficiency change

gml^{BPC} ———— $gml^{BPC\times EC}$ - - - - - -

Notes: All figures contain densities estimated using kernel density estimation for the different components of the bipartite decomposition in expression (11), considering the good and bad output orientation. The vertical lines in each plot represent the average for each component of the decomposition. Densities were estimated using a Gaussian kernel and the Sheather and Jones (1991) plug-in bandwidth.

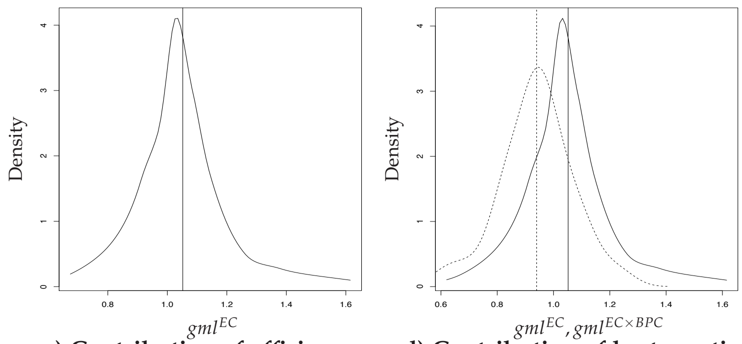
Figure 3: Kernel density plots of the bipartite decomposition of educational improvement, good and bad output orientation (local bandwidth)

Good and bad output orientation



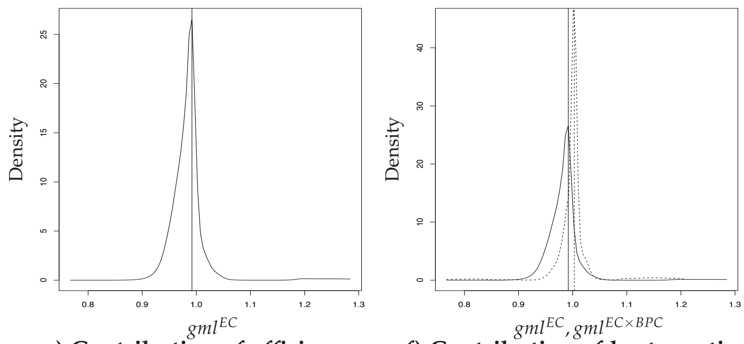
a) Contribution of efficiency change **b) Contribution of best practice gap change**

Good output orientation



c) Contribution of efficiency change **d) Contribution of best practice gap change**

Bad output orientation



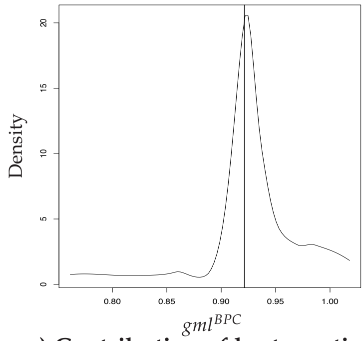
e) Contribution of efficiency change **f) Contribution of best practice gap change**

gml^{EC} ———— $gml^{EC\times BPC}$ _ _ _ _ _ _

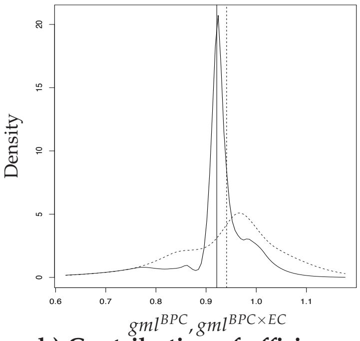
Notes: All figures contain densities estimated using kernel density estimation for the different components of the bipartite decomposition in expression (11), considering the good and bad output orientation. The vertical lines in each plot represent the average for each component of the decomposition. Densities were estimated using local likelihood methods (Loader, 1996), and a Gaussian kernel was chosen.

Figure 4: Kernel density plots of the bipartite decomposition of educational improvement, good and bad output orientation (local bandwidth)

Good and bad output orientation

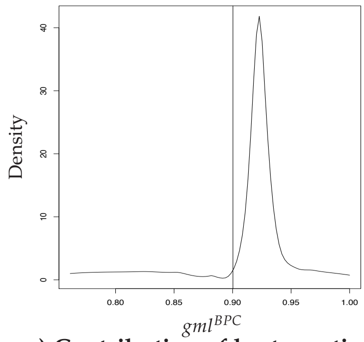


a) Contribution of best practice gap change

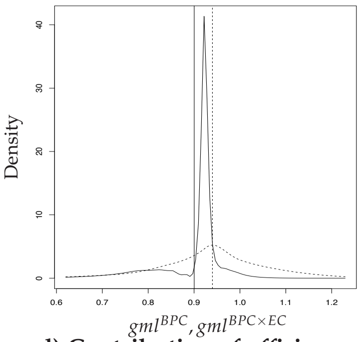


b) Contribution of efficiency change

Good output orientation

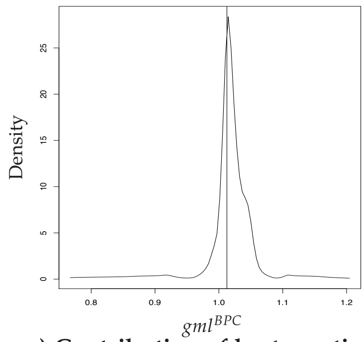


c) Contribution of best practice gap change

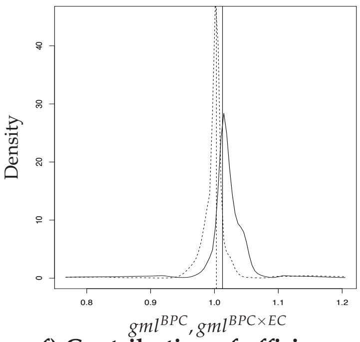


d) Contribution of efficiency change

Bad output orientation



e) Contribution of best practice gap change



f) Contribution of efficiency change

gml^{BPC} ——— $gml^{BPC\times EC}$ - - - - - -

Notes: All figures contain densities estimated using kernel density estimation for the different components of the bipartite decomposition in expression (11), considering the good and bad output orientation. The vertical lines in each plot represent the average for each component of the decomposition. Densities were estimated using local likelihood methods (Loader, 1996), and a Gaussian kernel was chosen.