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**Abstract**

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November 9, 2016

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# 1 Introduction

Banks are the main link between financial markets and real economy. They should provide financing to the private sector and pursue economic growth. However, in recent decades, they have transferred a massive amount of resources from the productive to the financial sector. This spectacular expansion and the growing influence of the financial sector do not seem to have promoted the economy, which is instead affected by recurrent crises (see Orhangazi 2008 and Rochon and Rossi 2010). Yet global banking linkages are viewed as having spread the profound difficulties due to the financial crisis that began in industrialized countries in 2007 (see, for instance, Fasika and Pozo, 2008; Kalemli-Ozcan et al. 2013; Grilli et al. 2015, 2015b). Such considerations prompted the International Monetary Fund's April 2009 World Economic Outlook (WEO) to argue that global bank linkages are "fueling the fire" of the current crisis (p. 149).

To understand what banks do when confronted with shocks becomes a key question for evaluating the stability of the economic system. Banks' strategies are not only a powerful channel of crisis transmission, but also an instrument of crisis prediction. On the one hand, by illustrating banks' responses to a liquidity shock, balance-sheet analysis explains the contagion propagation channel and the source of systemic risk. The idea is synthesized by the so-called "financial accelerator mechanism" (see Bernanke and Gertler 1990, 1995). An intuition of how this mechanism works is as follows. If one or more borrowers are not able to pay back their debts to the lender, the lender balance sheet decreases and, consequently, the borrowers' bad debt, which affects the equity of lender, can also lead to lender failures. As the lender may be a borrower in turn, the bad debt can thus bring about a cascade of bankruptcies. Furthermore, lenders, hit by bad debt, may seek to recover their losses by increasing the interest rate for other borrowers. This could lead to the failure of other highly leveraged borrowers, thus triggering a domino effect (see Delli Gatti et al. 2005 and Tedeschi et al. 2012). On the other hand, banks' behavior is a powerful instrument of crisis prediction. There is empirical evidence showing that big financial institutions are able to anticipate financial stress and act accordingly (see Martin et al. 2016).

Finally, financial institutions take advantage of private information on their counterparty insolvency risk and adjust loan contract terms in response to it (see Chen 2015).

Given the potentially large implications of banks' behavior for the real economy and financial markets, and with the current crisis in mind, this paper introduces an alternative approach to identifying banks' behavior in financial markets. In this regard, the economic literature identifies three ways[1]. The first is microfoundations, that is, the microeconomic analysis of the behavior of individual agents (see Stigler and Becker 1977, Blaug 1992 and Kirman 1992, among the first studies). The second is experimental economics, that is, the application of experimental methods to study how human subjects behave (see, among many, Hommes et al. 2005; Bao et al. 2013; Husler et al. 2013 and Agliari et al. 2016). And the third is agent-based economics, that is, the application of computational techniques to reproduce micro-/meso- and macro-dynamics via a bottom-up approach (see, Arthur 1993; Clark 1997; Tesfatsion and Judd 2006; Shoham et al. 2007).

Following this last line of research, this paper is explicitly concerned with the ability of agent-based models to describe micro behavior. In this regard, the agent-based literature mainly focuses on input validation, which ensures that the behavior of individual agents incorporated into the model captures the salient aspects of human subjects (see Tesfatsion 2013). The approach we follow in this paper is, instead, quite different. We start with a microfounded agent-based model, namely the well-known model developed by Brock and Hommes 1998 (BH hereinafter), and show how the calibrated values of the model parameters can describe real agents' behavior. Specifically, we focus on analyzing banks' behaviors, which are seen as a primary reason for the financial crisis.

It is important to recall that in the literature on the agent-based model calibration (see Chen et al. 2012 for a detailed review), the estimation of the BH model has a long tradition (see, among many, Boswijk et al 2007; de Jong et al. 2009, 2010; Manzan and Westerhoff 2007; Reitz and Sloppek 2009; Kukacka and Barunik 2016). Indeed, even if other types of financial agent-based models, ranging from ant-type models to Lux models, have been calibrated using different techniques, including the ordinary least squares and maximum likelihood methods (see, for instance, Alfarano et al. 2005; Gilli and Winker 2003; Chen and Lux 2015; Ghonghadze and Lux 2016), the strong dominance of models deriving from the BH adaptive belief system

[1]A discussion of these approaches is beyond the scope of this work. A detailed debate of these methodologies can be found in other studies (see, for instance, Kirman 2010).

is beyond doubt. This is due to its simplicity, its and the small number of parameters, all elements which make this model suitable for the empirical estimation.

Two steps are essential in achieving our goal. First, we have to prove that the BH model can successfully reproduce the daily price time series of the three banking indices investigated (i.e., the S&P SmallCap 600 Financials Index, the STOXX Europe 600 Banks, and the STOXX Asia/Pacific 600 Banks). This first step consists in verifying that our calibration procedure succeeds in validating the output. This analysis, briefly sketched in this work, is widely debated in Recchioni et al. 2015. In that paper, the authors show that the least squares calibration procedure that we use here enables the BH model to satisfactorily perform in descriptive and predictive output validations[2].

Second, having successfully proven our calibration for output validation, we can test its efficiency in describing agents' behavior. The key idea of our approach is simple: if the calibrated BH model reproduces the dynamics of the investigated markets well (i.e., step 1), one can infer that the values of its estimated parameters may be useful in understanding the evolution of the behavior of agents operating in those markets (i.e., step 2). This second step, which is the original contribution of the present paper, requires us to estimate the model parameters using rolling time windows for each of the three banking series. In this regard, two exercises are presented: a long-run analysis where the emphasis is placed on differences and similarities among the three different geographical areas (i.e., America, Europe, and Asia), and a short-run analysis where the focus is on the dynamic evolution of banks' strategies in response to the several economic phases characterizing the time series investigated. Specifically, in the long (**short**) run analysis, the model parameters are calibrated approximately every year (**two months**) using a year of past observations[3]. After each year (**two months**), we solve the calibration problem again, adding the 200 (**60**) new daily observations and discarding the 200 (**60**) oldest ones. In this way, the length of the time window used in the calibration is kept constant. Hence, we solve 26 (**90**) calibration problems and the solutions to these problems provide historical

[2]The calibration procedure proposed by Recchioni et al. 2015 identifies the set of model parameters by minimizing a loss function. This function is the sum of the squared residuals, which are computed as the difference between the observed and simulated market price on a given date. Moreover, as we do here, they solve the resulting minimization problem numerically via a gradient-based method (see, for example, Andersen and Andreasen 2000; Recchioni and Scoccia 2000).

[3]In our analysis, one year of data corresponds to 200 observations, while two months correspond to 60 observations.

series of each model parameter.

On the one hand, our long-run analysis shows the presence of strong similarities among the three areas. Indeed, in line with other studies (see Boswijk et al 2007, Recchioni et al. 2015), all three areas are characterized by the presence of collective behavior and the predominance of trend-follower behavior. The preponderance of the chartist strategy, with its well-known role in destabilizing prices, appropriately reflects the several financial crises characterizing the time series investigated (i.e., the years ranging from 1994 to 2016). Additionally, our results show that all countries are distinguished by very high levels in the risk-aversion parameter, in accordance with the registered financial turbulences.

On the other hand, our short-run findings suggest the key role of banks' strategic behavior in generating financial distress. In fact, we show that not only are price bubbles generated by a high percentage of chartist traders, but financial collapses are also heralded by a sharp increase in the number of these agents. Last but not least, our results show that when the crisis is in place, trend followers gradually lose their power and fundamentalists are able to drive prices to the fundamental value.

The remainder of the article is organized as follows. In Section 2, we describe the BH model and present the calibration technique. In Section 3, we present numerical experiments on the three banking indices. Finally, Section 4 presents our conclusions.

# 2 Model description and calibration technique

In this section, we first briefly describe the Brock and Hommes model (BH hereinafter) and then we illustrate the technique used for the calibration.

## 2.1 The Brock and Hommes model

The BH model, which uses the familiar demand-supply cobweb framework, considers an adaptive belief system where heterogeneous agents can choose from among different trading strategies. Two typical investors are distinguished: fundamentalists, who believe that the market price is completely determined by the fundamental value, and chartists, who think that the price can be predicted using some information about the past. The model then introduces evolutionary competition between the two strategies. At each time step, agents can revise their strategy according to a fitness measure based on accumulated past profits. The rewind algorithm is designed so that the strategy with the highest fitness gains a higher number of followers. Nonetheless, the algorithm introduces a certain amount of randomness, and the more successful strategy has a finite probability of not being followed. In this way, imperfect information and the bounded rationality of agents is reproduced. The randomness also helps to unlock the system from the situation where all agents follow the same strategy. The algorithm generates an endogenous switching between the two behaviors. This creates alternating periods: times when the different strategies co-exist and compete for popularity, and times when one of the two behaviors prevails and dominates the market. This alternation is the leading mechanism in generating temporary speculative bubbles.

In what follows, we describe the basic ingredients of the model. For more technical details we refer the reader to Brock and Hommes 1998 and Hommes 2001.

Agents, which are assumed to be myopic mean-variance maximizers, determine their demand for the risk asset via wealth maximization. The optimal level of risky share, $z_{h,t} = E_{h,t}(\bar{p}_{t+1} + y_{t+1} - R\bar{p}_t)/(\alpha\sigma^2)$, is a function of the expected market equilibrium price, $\bar{p}_{t+1}$, the dividend, $y_{t+1}$, the gross return of the risk free asset, $R = (1 + r) > 1$, the risk aversion, $\alpha$, and the conditional variance, $\sigma^2$. By imposing a zero supply of outside shares and different trader types, $h$, we use the optimal level of risky share to derive the market equilibrium price $\bar{p}_t = \frac{1}{R}\sum_{h=1}^{H} n_{h,t}E_{h,t}(\bar{p}_{t+1} + y_{t+1})$, where $n_{h,t}$ denotes the fraction of agents $h$ at time $t$. Moreover, in the case of identical and rational agents, we obtain the fundamental price, $p^*_t = \frac{1}{R}E_t(p^*_{t+1} + y_{t+1})$, from the market equilibrium price. In order to calculate the equilibrium and fundamental prices, agents must form their own expectations, $E_{h,t}$, on future prices and dividends. To this end, we assume that all beliefs are of the form $E_{h,t}(\bar{p}_{t+1} + y_{t+1}) = E_t(p^*_{t+1} + y_{t+1}) + f_h(\bar{x}_{t-1}, ..., \bar{x}_{t-L})$, where $\bar{x}_t = \bar{p}_t - p^*_t$ is the deviation of the price from the fundamental price and $L$ is the number of lags. It is evident that agents believe that market and fundamental prices may not coincide due to some function $f_h$ that depends on the past deviation from $p^*_t$. We assume the simplest version of the model with only one lag and two simple linear trading rules. The first one describes fundamentalists, i.e., $f_{h,t} = 0$, who believe that the market price will be equal to the fundamental price, or, equivalently, that the deviation, $\bar{x}_t$ from the fundamental price will be 0. The second strategy describes chartists, i.e., $f_{h,t} = g\bar{x}_{t-1}$, where $g$ is the trend parameter.

For each trading day, agents update their strategy and as a consequence, the fraction, $n_{h,t}$, of investor types evolves over time. This dynamics is governed by an endogenous fitness given by $U_{h,t} = (\bar{p}_t + y_t - R\bar{p}_{t-1})z_{h,t} + \omega U_{h,t-1}$, where $\omega \in [0,1]$ is a memory parameter[4]. Each agent starts with a strategy and computes its profitability with respect to the other one. A 'Gibbs' equation, equal to $n_{h,t} = \frac{\exp(\beta U_{h,t})}{\sum_{h=1}^{2} \exp(\beta U_{h,t})}$, then determines the probability that the trader switches from its own strategy to the other one. This, in turn, modifies the next equilibrium market price and all the other model dynamics.

## 2.2 The calibration technique

We illustrate the calibration technique applied to validate the BH model. As already stressed, we choose the BH model because of its tractability and the immediate interpretation of the parameters in terms of behavioral attitudes. The simplicity of the model allows us to easily determine differences and similarities among the financial markets analyzed during different stages of the economic cycle.

We introduce the key ingredients of the calibration procedure:

- $p_t^o$ is the daily closing index from the real dataset. The time is $t = 0, 1, \ldots, \tau - 1$ with $\tau > 1$. Specifically, $t = 0$ and $t = \tau - 1$ are, respectively, the first and the last observation dates used in the calibration procedure.
- $p_{h,t} = E_{h,t}(\bar{p}_{t+1})$, $t > 0$, $h = 1, 2$, is the agent expectation of the price at time $t$, $\bar{p}_t$, $t > 0$.
- $\bar{p}_t$, $t > 0$, is the simulated equilibrium market price at time $t$.
- $p_t^*$ is the fundamental price.
- $\bar{x}_t = \bar{p}_t - p_t^*$, $t > 0$, is the deviation from the fundamental price.

The calibration technique is composed of the following time steps:

Step $i_1$): compute the expectation of the price for fundamentalists, $h = f$, and chartists, $h = c$:

$$f_{f,t} = 0, \tag{1}$$
$$f_{c,t} = g\,\bar{x}_{t-1}. \tag{2}$$

[4]In a more complete version of the model, the cost of obtaining a "good" forecasting strategy is included in the fitness measure.

Step $i_2$): compute the agents' fitness measures:

$$U_{f,t-1} = [\bar{x}_{t-1} - R\bar{x}_{t-2}] \frac{(-R\bar{x}_{t-2})}{\alpha\sigma^2} + \omega\, U_{f,t-2}, \tag{3}$$

$$U_{c,t-1} = [\bar{x}_{t-1} - R\,\bar{x}_{t-2}] \frac{(g\bar{x}_{t-3} - R\,\bar{x}_{t-2})}{\alpha\sigma^2} + \omega\, U_{c,t-2}. \tag{4}$$

Step $i_3$): compute the simulated equilibrium market price and its deviation from the fundamental price:

$$\begin{aligned} \bar{x}_t &= (n_{f,t-1} f_{f,t} + n_{c,t-1} f_{c,t})/(1+r), \\ \bar{p}_t &= p_t^* + \bar{x}_t, \end{aligned} \tag{5}$$

where $n_{f,t}$, $n_{c,t}$ are given by:

$$n_{h,t-1} = \frac{\exp(\beta\, U_{h,t-1})}{\sum_{h=1}^{2} \exp(\beta U_{h,t-1})}, \quad h = f, c. \tag{6}$$

Step $i_4$) if $t \leq \tau$ go to Step $i_1$ else stop.

We underline that in the time window of the calibration procedure, we assume the dividend process, $y_t$, to be constant. Constant dividends imply a constant fundamental price. This assumption makes the proposed calibration process deterministic since it does not involve any noise in the previous steps.

We now formulate the calibration problems.

Let $\mathbb{R}^4$ be the four-dimensional real Euclidean space and $\underline{\Phi} \in \mathbb{R}^4$ be the vector containing the model parameters whose values have to be computed $\underline{\Phi} = (\alpha, p^*, \beta, g) \in \mathbb{R}^4$. Let $\mathcal{M} \subset \mathbb{R}^4$ be the set of the feasible parameter vectors defined as

$$\mathcal{M} = \left\{ \underline{\Phi} = (\alpha, p^*, \beta, g) \in \mathbb{R}^4 \,, \alpha \geq 0,\ \beta \geq 0 \right\} . \tag{7}$$

The calibration problem considered is formulated as follows:

$$\min_{\underline{\Phi} \in \mathcal{M}} F(\underline{\Phi}), \tag{8}$$

where the objective function $F(\underline{\Phi})$ is given by

$$F(\underline{\Phi}) = \sum_{t=1}^{\tau} \left( \frac{\bar{p}_t - p_t^o}{p_t^o} \right)^2 ,\ \underline{\Phi} \in \mathcal{M}. \tag{9}$$

The constrained optimization problem is solved via a metric variable steepest descent method (see Recchioni and Scoccia 2000). This is an iterative procedure that, given an initial point $\underline{\Phi}^0 \in \mathcal{M}$ and making steps in the direction of minus the gradient of $F$ in a suitable metric, generates a sequence $\{\underline{\Phi}^k\}$, $k = 0, 1, \ldots$, of feasible vectors (i.e., $\underline{\Phi}^k \in \mathcal{M}$, $k = 0, 1, \ldots$). The gradient is computed in a suitable metric defined according to the constraints defined in $\mathcal{M}$ and rescaled in order to ensure the convergence of the iterative process. The algorithm stops when a maximum number of iterations, $M_{iter}$, is performed or the Euclidean distance $||\underline{\Phi}^{k+1} - \underline{\Phi}^k||$ is less than a preassigned tolerance.

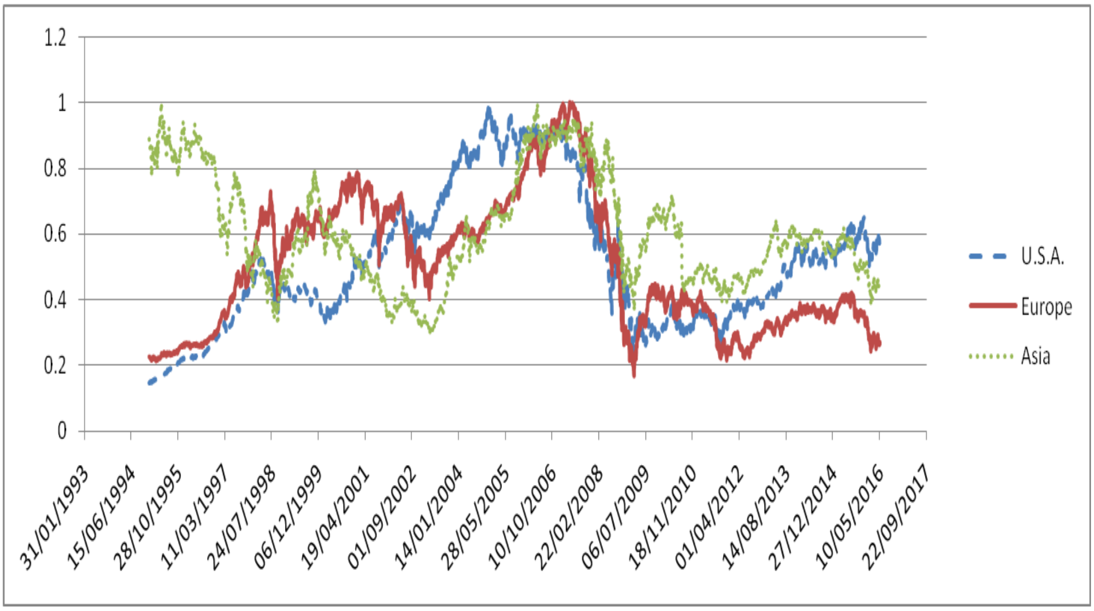

Figure 1: Re-scaled index values from December 30, 1994 ($t = 1$) to May 18, 2016. S&P SmallCap 600 Financials Index (blue dashed line), STOXX Europe 600 Banks (red solid line), and STOXX Asia/Pacific 600 Banks (green dotted line).

# 3 The calibration procedure at work

## 3.1 Description of the data

In this experiment, we calibrate the parameters of the BH model on three banking sectoral indices representing different geographical macro areas (i.e., USA, Europe, and Asia). We use the daily closing values of the S&P Small-Cap 600 Financials Index, the STOXX Europe 600 Banks, and the STOXX Asia/Pacific 600 Banks. The data run from December 30, 1994 to May 18, 2016, corresponding to 5579 daily observations.

Figure 1 shows the re-scaled index values used in the calibration exercise. The re-scaled observed market prices are obtained simply by dividing each index value by its maximum value over the entire time period considered. The analyzed time series consider different phases of boom and burst that have affected financial markets in the last twenty years. Specifically, the following episodes can be mentioned: first, the *mini crash* on October 27 1997, caused by an economic crisis in Asia that then propagated on the US and EU markets; second, the *World Trade center attack* on September 11 2001, which caused a sharp drop in global stock markets; third, the *internet bubble bust* between July and September 2002, which resulted in a dramatic decline in stock prices across the United States, Canada, Asia, and Europe; four, the *Lehman Brothers bankruptcy* on September 15, 2008, resulting in an abrupt collapse of all series; and, finally, a phase beginning with the outbreak of the *sovereign debt crisis* on May 2010.

Figure 1 also shows a strong co-movement in the time series behavior, which confirms a strong interconnection between financial systems. This has been especially true since 2003, with a clear anticipation of the US economy on the European and Asian economies.

In order to calibrate our model, the choice of the starting point, $\underline{\Phi}^0$ must be made carefully. This problem is handled by calculating the best value of the objective function (see Eqs. 8–9) on a set of random initial points belonging to the feasible region $\mathcal{M}$.

Table 1: Initial points of the BH calibration procedure.

| **Parameter** | **US** | **Europe** | **Asia/Pacific** |
|---|---|---|---|
| $\beta$ | 0.6 | 1.5 | 0.6 |
| g | 2.0 | 2.0 | 2.0 |
| $p^*_S$ | 0.45 | 0.43 | 0.47 |
| $\alpha$ | 19.2 | 19.2 | 18.2 |
| $\omega$ | 1 | 1 | 1 |

Table 1 shows the parameter values corresponding to the smallest values of the objective functions (i.e., our starting points in the calibration exercise). By comparing the initial points for the three banking sectoral indices, it is worth noting that they are quite similar, with the only exception being for $\beta$ in the European market and $\alpha$ in the Eastern market. However, the most striking result is the absolute coincidence in the values of the parameters $g$ and $\omega$.

Firstly, it is important to note that parameter $g$ describes the trend-follower behavior. A value of $g$ close to 2 has also been found in other studies involving different market indices (see Boswijk et al. 2007 and Recchioni et al. 2015). This suggests that the persistence of the trend-following strategy and its ability to deviate prices from the fundamental price is a constant feature of financial markets. It is worth noting that when $g > 2(1+r)$, the simulated prices diverge from the fundamental price and move to other basins of attraction. The divergency is mathematically due to the fact that $n_{f,t} \approx n_{c,t} \approx 1/2$ for $t > t_0 > 0$ so that from Eq. 5 we have $\bar{x}_t = (\frac{g}{2(1+r)})^{t-t_0}\,\bar{x}_{t_0}$, $t > t_0$. This implies that the simulated equilibrium price does not converge to $p^*$ as $t \to +\infty$ when $g > 2(1+r)$, while it does converge to $p^*$ when $g < 2(1+r)$. Our estimation procedure probably forces parameter $g$ to converge to this threshold from below in order to have a longer transient period before converging to the fundamental price. During this long transient period, the trajectory is flexible and provides a good fit to the observed data.

Secondly, it is important to note that parameter $\omega$ refers to agent memory. When $\omega$ is zero, the model reproduces a situation with no memory, that is, the fitness equals the realized profit in the previous period. Otherwise, when $\omega = 1$, the model generates a situation with infinite memory, that is, the fitness equals the total wealth as given by accumulated realized profits over the entire past. For this parameter there is also empirical evidence proving the presence of high memory in agent fitness (see Recchioni et al. 2015). The presence of high memory also has an important theoretical consequence. It is not clear whether the price dynamics are stable in the case of infinite memory (see Hommes, 2001). Specifically, it still an open question whether, in this circumstance, fundamentalists are able to stabilize the price towards its fundamental value and can drive trend-followers out of the market (see Brock and Hommes 1998; Hommes 2001).

## 3.2 From the estimation of model parameters to the identification of banks' strategies

In this section we investigate the ability of the calibrated model parameters to describe banks' behavior. In fact, the values of parameters resulting from calibration of the model on different markets can show differences and similarities in the behavior of agents operating in the markets considered. Two applications are considered. The first is a long-run analysis among countries (application (a)). Here the purpose is to describe the relations among the different geographical areas. In this first exercise we calibrate the parameters on long time windows (i.e., 200 steps each), and the final values representing the banks' strategies are obtained as an average over the whole time series.

The second application tries to identify how banks' strategies evolve over time (application (b)). The idea is to understand how banks react during different economic phases. In this second exercise, we calibrate the parameters on short time windows (i.e., 60 steps each). By generating time series of the optimal value of the calibrated parameters, this process allows us to investigate how banks dynamically change their behavior in regard to financial turmoils. The two exercises are particularly interesting because they allow us to analyze banks' behavior not only with respect to the geographic areas of interest, but also with respect to different economic phases.

Before analyzing the ability of the calibrated parameters to describe banks' strategies, we must test the ability of the calibration technique to reproduce the empirical data. To this end, we show that the model is capable of reproducing the daily price time series of the three different indices.

In this first exercise, we solve problem (8) with $\tau = 200$ and $M_{iter} = 1000$. In the calibration procedure, we fix parameters $\sigma = 0.1$, $r = 0.01/250$ (daily risk free return), and $\omega = 1$. The initial point of the fraction $n_{h,0}$ is 0.5. The initial values of the remaining parameters are fixed as in Table 1. Table 2 shows the optimal values of the model parameters obtained from the calibration procedure. Since the calibration procedure is deterministic (i.e., it does not include any noise in the simulated equilibrium market price), the confidence interval of the estimated values of the parameters in Tables 2 are obtained by running the calibration procedure on 100 trajectories for each index. These trajectories are obtained by perturbing each index by adding a noise sampled from a normal distribution with zero mean and standard deviation given by $\sigma_n = \xi\sigma_s$, where $\sigma_s$ is the standard deviation of the observed data and $\xi$ is a constant equal to 1%. Standard deviations (St.Dev.), mean relative errors (Rel.Err.) and biases confirm that the parameters are statistically significant. Moreover, in order to assess the accuracy of our method, for each index $r$, we calculate the relative errors of the simulated equilibrium market prices. Specifically, Fig. 2 shows the quantities $e_{r,t} = |p_t^o - \bar{p}_t||/|p_t^o|$. We observe that, on average, the relative error is 0.00726. This indicates that the simulated market prices match the observed prices even when they are affected by abrupt changes.

Having successfully proven our calibration for descriptive output validation, we can test its efficiency in describing banks' strategies. The results regarding application (a) (i.e., the intra-countries analysis) are already contained in Tab. 2. In fact, the table provides information on differences and similarities in banks' behavior operating in the different geographical areas.

Table 2: Model parameter and objective function values obtained with the calibration procedure.

| **Parameters** | **S&P Financials Index** | **STOXX Europe** | **STOXX Asia/Pacific** |
|---|---|---|---|
| $\beta$ | 1.64 | 1.75 | 1.56 |
| St. Dev | ($7.500 \cdot 10^{-4}$) | ($3.682 \cdot 10^{-4}$) | ($1.304 \cdot 10^{-3}$) |
| Rel. Err. | ($1.457 \cdot 10^{-3}$) | ($4.478 \cdot 10^{-2}$) | ($9.778 \cdot 10^{-1}$) |
| Bias | ($6.645 \cdot 10^{-3}$) | ($2.333 \cdot 10^{-3}$) | ($7.278 \cdot 10^{-3}$) |
| g | 1.95 | 1.96 | 2.00 |
| St. Dev | ($3.759 \cdot 10^{-4}$) | ($8.496 \cdot 10^{-3}$) | ($5.556 \cdot 10^{-2}$) |
| Rel. Err. | ($3.721 \cdot 10^{-3}$) | ($1.991 \cdot 10^{-3}$) | ($2.147 \cdot 10^{-3}$) |
| Bias | ($2.670 \cdot 10^{-3}$) | ($4.483 \cdot 10^{-4}$) | ($-4.567 \cdot 10^{-4}$) |
| $\alpha$ | 423.21 | 254.56 | 344.03 |
| St. Dev | ($1.413 \cdot 10^{-1}$) | ($5.54 \cdot 10^{-1}$) | ($1.167 \cdot 10^{-1}$) |
| Rel. Err. | ($1.875 \cdot 10^{-2}$) | ($2.719 \cdot 10^{-4}$) | ($4.986 \cdot 10^{-5}$) |
| Bias | ($-6.178 \cdot 10^{-2}$) | ($2.743 \cdot 10^{-3}$) | ($-4.676 \cdot 10^{-4}$) |
| $p_S^*$ | 459.2403 (0.29) | 537.725 (0.28) | 104.024 (0.29) |
| St. Dev | ($3.266 \cdot 10^{-3}$) | ($7.913 \cdot 10^{-3}$) | ($6.957 \cdot 10^{-3}$) |
| Rel. Err. | ($4.970 \cdot 10^{-3}$) | ($1.121 \cdot 10^{-2}$) | ($1.987 \cdot 10^{-3}$) |
| Bias | ($5.327 \cdot 10^{-3}$) | ($7.028 \cdot 10^{-3}$) | ($1.216 \cdot 10^{-3}$) |
| $F_{BH}(\underline{\Phi}^*)$ | 0.00176 | 0.0022 | 0.0032 |
| St. Dev | ($6.956 \cdot 10^{-5}$) | ($1.878 \cdot 10^{-4}$) | ($1.492 \cdot 10^{-4}$) |
| Rel. Err. | ($4.738 \cdot 10^{-2}$) | ($6.435 \cdot 10^{-2}$) | ($1.986 \cdot 10^{-2}$) |
| Bias | ($2.432 \cdot 10^{-4}$) | ($3.784 \cdot 10^{-4}$) | ($1.061 \cdot 10^{-4}$) |

Specifically, Table 2 describes banks' imitative behavior $\beta$, risk aversion $\alpha$, trend follower behavior $g$, and fundamental values $p^*$. It is interesting to note that all three areas are characterized by similar values of $\beta$ and $g$. This indicates the presence of collective behavior and the predominance of trend follower behavior. It is well known in agent-based literature that mechanisms of behavioral switching and collective behavior, emerging in situations with information externalities, can lead to large aggregate fluctuations (see, for instance, Lux and Marchesi 2000; Chiarella et al. 2009; Kirman and Teyssiere 2002; LeBaron and Yamamoto 2009; Tedeschi et al. 2012). This result is in line with the many bubble and crash episodes that have occurred over the last thirty years. Moreover, the high values of risk aversion characterizing all markets further support the existence of a strong instability in the investigated time series. It is worth noting that by removing the speculative bubble episodes from the investigated indices, the average risk aversion sharply decreases and reaches the values of 20.33 (st.dev $1.1413 \cdot 10^{-1}$), 19.35 (st.dev

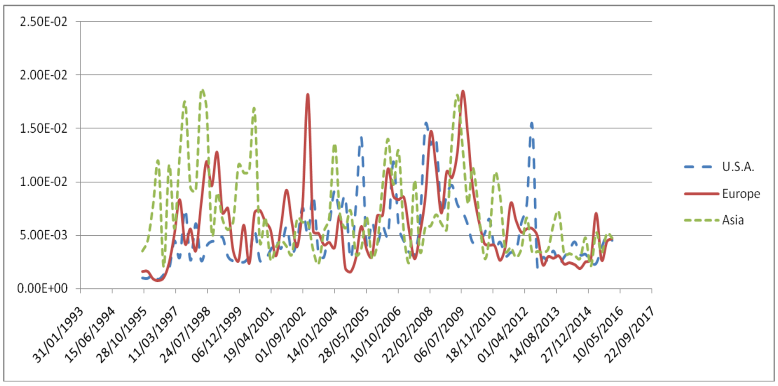


Figure 2: Relative errors, $e_{r,t}$, between the observed price and simulated equilibrium market price versus time, obtained using the BH model with the parameters shown in Table 2. S&P SmallCap 600 Financials Index (blue dashed line), STOXX Europe 600 Banks (red solid line), and STOXX Asia/Pacific 600 Banks (green dotted line).

$5.54{\cdot}10^{-1}$), and 17.49 (st.dev $1.167{\cdot}10^{-1}$) for the US, EU, and Asian indices respectively. Finally, Table 2 provides some insight on fundamental prices. In agreement with the empirical evidence attesting that developing markets have lower fundamental prices, the value of $p^*$ in the Asian market is almost five time smaller than in the two Western markets.

We now investigate the reaction of banks when faced with financial turbulence in the last twenty years (i.e., application (b)). To this end, we solve problem (8) with $\tau = 60$. Specifically, the model parameters are calibrated approximately every two months. After two months, we solve the calibration problem again adding the 60 new daily observations and discarding the 60 oldest ones. In this manner, we solve 90 calibration problems, whose solutions generate time series for each model parameter. All remaining parameters are those used in the previous calibration experiment.

In order to assess the robustness of the estimation procedure, we evaluate the sensitivity of the estimated parameter values with respect to the number of observations, $\tau$, used in the time window. Specifically, we compare the values of application (b) with those obtained in application (a) with $\tau = 200$ by applying the two-sample Kolmogorov-Smirnov (KS) goodness-of-fit hypothesis test. The test confirms that the historical series of model parameters, estimated using the two samples, are drawn from the same population at a significance level of 5%.

Figure 3 shows the time series of the estimated parameters for the three indices. A noteworthy feature is the strong volatility in the model parame-

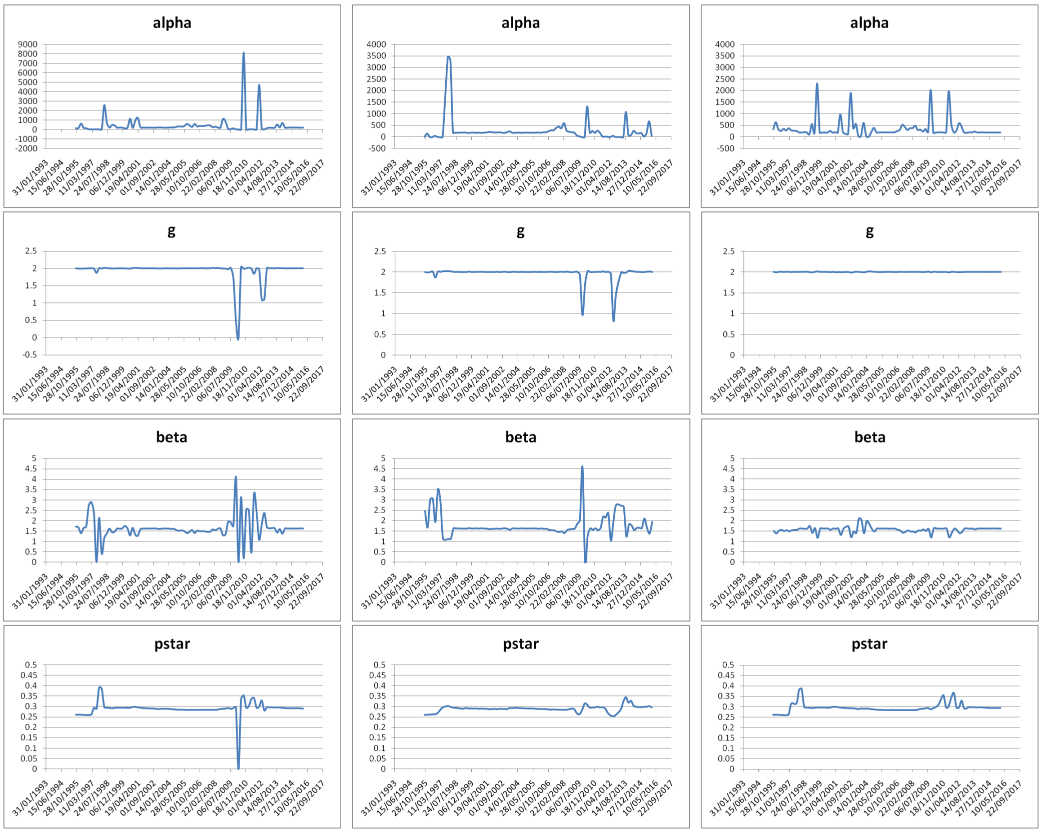


Figure 3: Risk aversion (top row), trend follower behavior (second row), imitative behavior (third row), and fundamental price (bottom row) time series for the S&P SmallCap 600 Financials Index (first column), the STOXX Europe 600 Banks (second column), and the STOXX Asia/Pacific 600 Banks (third column).

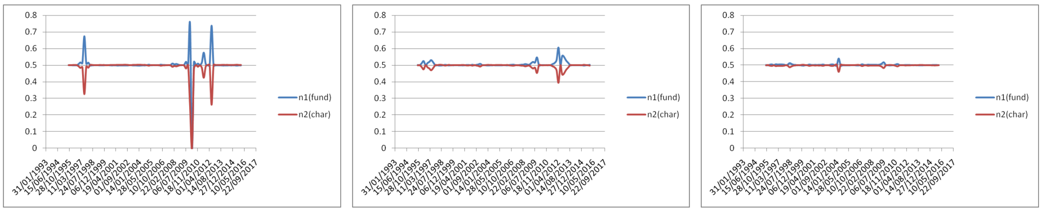


Figure 4: Time series of the fraction of fundamentalists, $n_f$, and trend-followers, $n_c$, for the S&P SmallCap 600 Financials Index (left panel), the STOXX Europe 600 Banks (central panel), and the STOXX Asia/Pacific 600 Banks (right panel).

ters emerging during periods of financial instability. The episodes of financial turmoil are reflected in the dynamics of all parameters, although they are particularly evident for the Western countries (see first and second columns of Fig. 3). Specifically, we observe a sharp rise in risk aversion, $\alpha$, related to financial stress events. Parameter $\alpha$ strongly increases in all three geographical areas during the 1998 Asian financial crisis and the 2008 Lehman Brothers collapse. By analyzing the risk aversion time series of the American index (see Fig. 3, first column), we recognize other important moments of instability, such as the Twin Towers attack in 2001, the downgrading of the United States' credit rating by the credit rating agency Standard & Poor's in 2011, and the 2013 US debt-ceiling crisis. Similar geographically localized episodes can be seen in the dynamics of the European and Asian $\alpha$ parameters. On the one hand, the European risk aversion strongly resents the sovereign debt crisis starting in 2010 (see second column of Fig. 3). On the other hand, the Asian parameter shows the 2002 internet bubble burst, the 2009 fall in GDP and exports, and the two tsunamis in 2004 and 2011 (see third column of Fig. 3).

The second row of Fig. 3 displays the time evolution of trend follower behavior. As for the long-term analysis (see application (a) above), the optimum value of parameter $g$ is always around two. In accordance with other studies (see Boswijk et al 2007, Recchioni et al. 2015), this confirms the predominance of the chartist strategy and its impact in destabilizing prices. It is important to mention that chartists are investors believing that price movements can be predicted by studying past trends. Specifically, they use historical price time series to forecast future trends. In this respect, a question arises: does this strategy make sense during periods of high instability? Our empirical analysis responds negatively to this question. In fact, we detect a negative correlation between the time series of the risk aversion $\alpha$ at time $t$ and the trend follower behavior $g$ at time $t+2$. Correlation values are $-0.622$, $-0.253$, and $-0.301$ for the American, European, and Asian indices, respectively. This result indicates that after prolonged financial tensions highlighted by high riskiness, investors do not rely on information coming from historical time series and, therefore, decrease $g$.

The dynamics of banks' imitative behavior, $\beta$, is shown in the third row of Fig. 3. This parameter, also known as intensity of choice, answers the question of how much agents trust information about other banks' performance. By multiplying and then amplifying the fitness measure, $U_h$, in the Gibbs equation (6), $\beta$ is a key parameter in determining the fraction of banks, $n_h$, that follow the chartist or fundamentalist strategy. A value of $\beta$ equal to zero shows complete lack of confidence in other agents' performance, while a high value of the parameter reflects a high level of trust in the success of other banks' strategies. The evolution of the parameter in the two Western markets (see first and second column) shows the presence of volatility clustering, a phenomenon reflecting the transition from quiet periods to turbulent ones. Specifically, during the 1997 Asian financial crisis and the 2008 global financial crisis, we observe erratic behavior in the parameter dynamics. This reflects a well-known phenomenon: during episodes of great uncertainty, economic agents respond chaotically, making their choices unpredictable (see Arthur 1994 and Behrens et al. 2007). The evolution of the parameter in the Asian market instead behaves differently (see third column). Indeed, it is affected by the time series instability, but its volatility is considerably lower. Specifically, in this market, the presence of volatility clustering is not observed[5]. It would seem that Asian banks are less exposed to financial instability phenomena. This observation is confirmed by some empirical studies (see Goldstein and Xie 2009) showing that Asia is protected by its low exposure to US subprime loans and securities, ample international reserves, current account surpluses, low dependence on commodity exports, a high share of interregional trade, improved banking systems, and an ability to implement countercyclical macroeconomic policies.

Finally, the last row of Fig. 3 displays the time evolution of fundamental prices. As mentioned above, in the calibration exercise, the fundamental price is treated as a constant unknown model parameter which is estimated over the investigated time window. As the reader can see, the fundamental price is also affected by economic turbulence. However, as expected, it remains constant during quiet periods.

We now investigate the switching phenomenon in agents' strategies. The issues are why traders change behavior and the impact of switching on the price time series. Fig. 4 displays the fraction of traders, $n_{h,t}$, following the fundamentalist or chartist strategy. The figure shows time variations between the two predictors, and this is more evident in the two Western markets. This result is in line with other empirical studies (see Boswijk et al. 2007 and Recchioni et al. 2015) showing the ability of the BH model to create behavioral switching in agents' strategies.

An important assumption of the BH model is the mechanism of belief formation, which follows the form $E_{h,t}(\bar{p}_{t+1} + y_{t+1}) = E_t(p^*_{t+1} + y_{t+1}) +$

[5]In accordance with the empirical literature (see Cont 2001 and Tedeschi et al. 2009), we check if volatility is persistent by measuring the autocorrelation function of absolute $\beta$ for different time lags. We note that while the autocorrelation of the parameter is insignificant in the Asian market, a positive and slowly decaying autocorrelation of absolute $\beta$ is present in the two Western markets. In this case, the autocorrelation function of absolute parameters is well fitted by a power law with exponent equal to 2.7 and 3.1 for the US and EU time series respectively.

$f_h(\bar{x}_{t-1}, ..., \bar{x}_{t-L})$. This assumption implies that investors are encouraged to follow the fundamentalist strategy when they observe market prices close to the fundamental price. In order to verify the reasons driving investors to choose this strategy, we analyze the correlation between the fraction of fundamentalists and the distance between the market price and the fundamental price. Specifically, is it correct to think that a price realignment toward its fundamental level induces agents to become fundamentalists? The correlations between $n_{f,t}$ and the differences between observed and fundamental price, $(p_t^o - p_t^*)$, are $-0.3343$, $-0.4632$, and $-0.2017$ for the US, EU, and Asian indices, respectively. The significant negative values of the correlations confirm the above assumption. It is important to emphasize that these values do not derive from a simple modeling assumption. In fact, they are derived using real market prices, $p_t^o$, and not the simulated market prices, $\bar{p}_t$. On the one hand, this result shows that the fundamentalist strategy emerges when the price is close to the fundamental price. On the other hand, the result further highlights the ability of the calibration technique to reproduce real prices.

Finally, we investigate the impact of agent beliefs on the price time series. The theoretical and empirical literature have shown the destabilizing effect of the chartist strategy on price dynamics (see, Boswijk et al. 2007; Chiarella et al. 2009; Recchioni et al. 2015 ). In line with this view, we show that price bubbles are generated by a high percentage of chartist traders. Table 3 shows the correlations between the index time series (see Fig. 1) and the lagged time series of $n_{c,t}$ (see Fig. 4). As can be seen, correlations increase at one-two

| *Correlation* | Lag $\tau = 0$ | Lag $\tau = 1$ | Lag $\tau = 2$ | Lag $\tau = 3$ | Lag $\tau = 4$ |
|---|---|---|---|---|---|
| $US_{\mathbf{f,t}}$ - $n_{\mathbf{c,t}-\tau}$ | 0.2148 | 0.2155 | 0.2509 | 0.2292 | 0.0028 |
| $EU_{\mathbf{f,t}}$ - $n_{\mathbf{c,t}-\tau}$ | 0.3683 | 0.3925 | 0.3069 | 0.1009 | 0.0498 |
| $Asia_{\mathbf{f,t}}$ - $n_{\mathbf{c,t}-\tau}$ | 0.2177 | 0.2769 | 0.0475 | 0.0357 | 0.0157 |

Table 3: Correlations between the index time series and the fraction of chartists, $n_{c,t-\tau}$, with $\tau$= 0; 1; 2; 3; 4 at a 1% confidence level.

lags (i.e., two-four months) before the abrupt change in the indices, and they decrease at zero lag. This result confirms that large aggregate fluctuations emerge as chartists take power. Moreover, given that the time series of $n_{c,t}$ is lagged, the fraction of chartists can be considered an “indicator” capable of anticipating the financial instability.

# 4 Conclusion

In this work, we have analyzed the ability of a well calibrated agent-based model to describe agents' strategic behavior through the value of the estimated parameters. By calibrating the BH model on daily data of three bank indices (i.e., US, EU, and Asian) running from 1994 to 2016, we have answered some questions regarding both similarities and differences in the behavior of banks operating in different geographical areas and the evolution of banks' strategies during several economic phases.

We have detected many similarities among the investigated areas. Specifically, all three markets are characterized by the presence of collective behavior and the predominance of trend follower behavior. Moreover, high values of risk aversion characterizing all markets further support the existence of a strong instability in the time series investigated. Our analysis, however, has also highlighted an important difference among markets. In fact, Western countries have appeared strongly perturbed by the financial instability affecting the periods considered. The parameters of these countries have shown volatility clustering indicating long transition periods between frenzied and calm times. Specifically, the erratic behavior of the intensity of choice parameter have shown this phenomenon and highlighted the incapacity of traders to cope with uncertainty.

With regard to the evolutionary analysis, our technique has shown some important features in banks' behavior. First, we have observed a decline in the power of the chartist strategy during crises. This indicates that prolonged financial tensions induce banks not to rely on information on past prices. Moreover, our results have shown the emergence of switching behaviors. On the one hand, we have noted that fundamentalists work as a thermostat of the society by realigning prices to the fundamental price. On the other hand, our analysis has clearly revealed the destabilizing power of chartists. These traders, in fact, not only generate asset bubbles, but also herald their arrival. In this respect, our study has shown that large aggregate fluctuations in the indices' time series are preceded by an increase in the number of trend-followers.

# Acknowledgements

The research leading to these results received funding from the European Union Seventh Framework Programme FP7 under grant agreement FinMaP no.: 612955.

# References

[1] Agliari, A., Hommes, CH., Pecora, N. 2016. Path dependent coordination of expectations in asset pricing experiments: A behavioral explanation. Journal of Economic Behavior and Organization,121(C),15-28.

[2] Alfarano, S., Lux, T., Wagner, F., 2005. Estimation of Agent-Based Models: The Case of an Asymmetric Herding Model. Computational Economics, 26 (1), 19-49.

[3] Andersen, L., Andreasen, J. 2000. Jump diffusion models: volatility smile fitting and numerical methods for pricing. Review of Derivatives Research, 4, 231-262.

[4] Arthur, W. B. (1994). Inductive reasoning and bounded rationality. The American economic review, 84(2), 406-411.

[5] Bao, T., Duffy, J., Hommes, C. 2013. Learning, forecasting and optimizing: An experimental study. European Economic Review, Elsevier, 61(C),186-204.

[6] Behrens, T. E., Woolrich, M. W., Walton, M. E., and Rushworth, M. F. (2007). Learning the value of information in an uncertain world. Nature neuroscience, 10(9), 1214-1221.

[7] Bernanke B., Gertler M., 1990. Financial fragility and economic performance. Q J Econ 105(1):87-114

[8] Bernanke B., Gertler, M. 1995. Inside the black box: the credit channel of monetary policy transmission. J Econ Perspect 9(4):27-48.

[9] Blaug, M. 1992. The Methodology of Economics: Or, How Economists Explain. Ed. Cambridge University Press.

[10] Boswijk, H.P., Hommes, C., Manzan, S., 2007. Behavioral heterogeneity in stock prices. Journal of Economic Dynamics and Control, 31(6), 1938-1970.

[11] Brock, W.A., Hommes, C.H. 1998. Heterogeneous beliefs and routes to chaos in a simple asset pricing model. Journal of Economic Dynamics and Control, 22, 1235-1274.

[12] Chen, S.H., Chang, C.L, Du, Y.R., 2012. Agent-based economic models and econometrics. The Knowledge Engineering Review 27, 187-219.

[13] Chen, PC. 2015. Banks' Acquisition of Private Information About Financial Misreporting. Accounting Review, Forthcoming. Available at SSRN: http://ssrn.com/abstract=2636066.

[14] Chen, Z. Lux, T., 2015. Estimation of sentiment effects in financial markets: A simulated method of moments approach. FinMaP-Working Paper 37, University of Kiel, Department of Economics, Kiel.

[15] Chiarella, C., Iori, G., Perello, J., 2009. The impact of heterogeneous trading rules on the limit order book and order flows. Journal of Economic Dynamics and Control 33 (3), 525-553.

[16] Clark, A. 1997. Economic Reason: The Interplay of Individual Learning and External Structure. in The Frontiers of the New Institutional Economics, Academic Press, Chapter XII (pp. 269-290).

[17] Cont, R. 2001. Empirical properties of asset returns: stylized facts and statistical issues. Quantitative Finance, Vol 1.

[18] Delli Gatti D, Guilmi C, Gaffeo E, Giulioni G, Gallegati M, 2005. A new approach to business fluctuations: heterogeneous interacting agents, scaling laws and financial fragility. Journal of Economic Behavior and Organization 56: 489-512.

[19] de Jong, E., Verschoor, W. F. C., Zwinkels, R. C. J., 2009. Behavioural heterogeneity and shift contagion: Evidence from the asian crisis. Journal of Economic Dynamics and Control 33, 1929-1944.

[20] de Jong, E., Verschoor, W. F. C., Zwinkels, R. C. J., 2010. Heterogeneity of agents and exchange rate dynamics: Evidence from the EMS. Journal of International Money and Finance 29, 1652-1669.

[21] Fasika, H., and Pozo, S. 2008. Currency crisis contagion and the identification of trasmition channels. International Review of Economics & Finance 17 (4), 572-588.

[22] Ghonghadze, J., Lux, T., 2015. Bringing an elementary agent-based model to the data: Estimation via GMM and an application to forecasting of asset price volatility. FinMaP-Working Paper 38, University of Kiel, Department of Economics, Kiel.

[23] Gilli, M., Winker, P., 2003. A global optimization heuristic for estimating agent based models. Computational Statistics and Data Analysis 42, 299-312.

[24] Grilli, R., Tedeschi, G. and Gallegati, M. 2015. Markets connectivity and financial contagion. J Econ Interact Coord 10: 287. doi:10.1007/s11403-014-0129-1.

[25] Grilli, R., Tedeschi, G. and Gallegati, M. 2015b. Bank interlinkages and macroeconomic stability. International Review of Economics & Finance 34, 72-88

[26] Goldstein, M., Xie, D. (2009). The impact of the financial crisis on emerging Asia. Peterson Institute for International Economics Working Paper, (09-11).

[27] Hommes, C., 2001. Financial markets as nonlinear adaptive evolutionary systems. Quantitative Finance, 1(1), 149-167.

[28] Hommes, C., Sonnemans, J., Tuinstra, J., van de Velden, H. 2005. Coordination of Expectations in Asset Pricing Experiments. Review of Financial Studies, 18(3), 955-980.

[29] Husler, A., Sornette, D., Hommes, C.H. 2013. Super-exponential bubbles in lab experiments: Evidence for anchoring over-optimistic expectations on price. Journal of Economic Behavior and Organization, 92(C), 304-316.

[30] Kalemli-Ozcan, S., Papaioannou, E., and Perri, F. 2013. Global banks and crisis transmission. Journal of international Economics, 89(2), 495-510.

[31] Kirman, A.P. 1992. Whom or What Does the Representative Individual Represent? Journal of Economic Perspectives, 6 (2): 117-136., doi:10.1257/jep.6.2.117.

[32] Kirman, A., Teyssiere, G., 2002. Microeconomic models for long-memory in the volatility of financial time series. Studies in Nonlinear Dynamics and Econometrics 5, 281.

[33] Kirman, A. 2010. Complex economics: individual and collective rationality. Ed. Routledge.

[34] Kukacka, J., Barunik, J., 2016. Estimation of Financial Agent-Based Models with Simulated Maximum Likelihood. IES Working Paper 7/2016. IES FSV. Charles University in Prague. Available at SSRN: https://ssrn.com/abstract=2783663.

[35] LeBaron, B., Yamamoto, R., 2009. The impact of imitation on long memory in an order-driven market. Eastern Economic Journal 34 (4), 504-517.

[36] Lux, T., Marchesi, M., 2000. Volatility clustering in financial markets: a micro simulation of interacting agents. International Journal of Theoretical and Applied Finance 3.

[37] Marin, JM., Akin, O., Peydro, JL. 2016. Anticipating the Financial Crisis: Evidence from Insider Trading in Banks. CEPR Discussion Paper No. DP11302. Available at SSRN: http://ssrn.com/abstract=2786904.

[38] Manzan, S., Westerhoff, F. H., 2007. Heterogeneous expectations, exchange rate dynamics and predictability. Journal of Economic Behavior and Organization 64, 111-128.

[39] Orhangazi, O., 2008. Financialization and capital accumulation in the non-financial corporate sector. Camb. J. Econ. 32(6), 863-886.

[40] Recchioni, M.C., Scoccia, A. 2000. A stochastic algorithm for constrained global optimization. Journal of Global Optimization, 16, 257-270.

[41] Recchioni, M.C., Tedeschi, G., Gallegati, M. 2015. A calibration procedure for analyzing stock price dynamics in an agent-based framework. Journal of Economic Dynamics and Control, 60, 1-25.

[42] Reitz, S., Slopek, U., 2009. Non-linear oil price dynamics: A tale of heterogeneous speculators? German Economic Review 10 (3), 270-283.

[43] Rochon LP, Rossi S 2010. Has It happened again? Int J Polit Econ 28(4):832-862.

[44] Shoham, Y., Powers, R., Grenager, T. 2007. If Multi-Agent Learning is the Answer, What is the Question? Artificial Intelligence 171, 365-377.

[45] Stigler, G., Becker, G. 1977. De gustibus non est disputandum. American Economic Review. 67 (2): 76.

[46] Tedeschi, G., Iori, G., Gallegati, M. 2009. The role of communication and imitation in limit order markets. European Physical Journal B, 1, 489-497.

[47] Tedeschi, G., Iori, G., Gallegati, M. 2012. Herding Effects in Order Driven Markets: The Rise and Fall of Gurus. J. Econ.Behav.Organisazion, 81, 82-96.

[48] Tedeschi G., Mazloumian, A., Gallegati, M., Helbing, D. 2012. Bankruptcy Cascades in Interbank Markets. PLoS ONE 7(12): e52749. doi:10.1371/journal.pone.0052749.

[49] Tesfatsion, L., Judd, K. L. (Eds.). 2006. Handbook of computational economics: agent-based computational economics (Vol. 2). Elsevier.

[50] Tesfatsion, L., 2013. Website on Validation of ACE: http://www.econ.iastate.edu/tesfatsi/empvalid.htm.